Flight Control Design Using Backstepping

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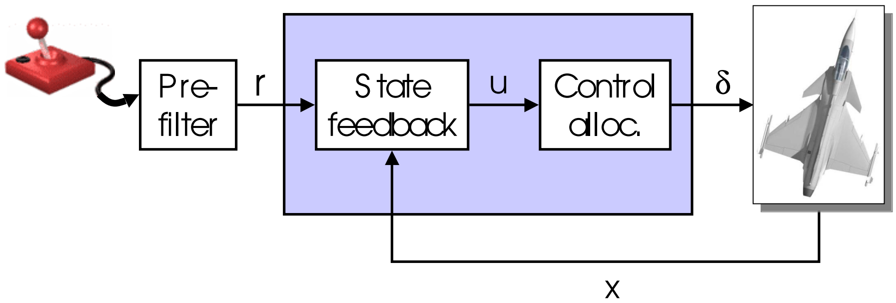
Background

? Can you design a single controller that will give stability and performance throughout the entire flight regime?

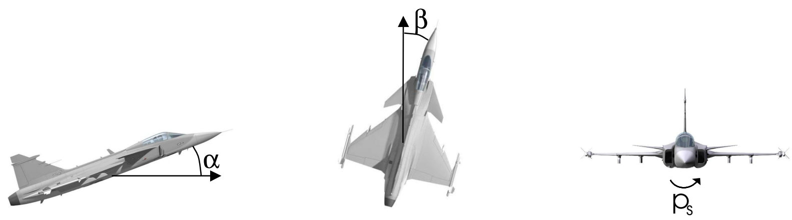
- Avoid tedious gain-scheduling.
- Previous work: feedback linearization.

New: Try backstepping!

Controller overview

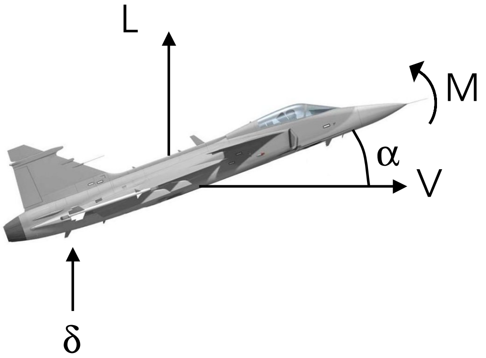


Control objectives



$\alpha = \alpha^{ref}$ $\quad$ $\beta = 0$ $\quad$ $p_s = p_s^{ref}$

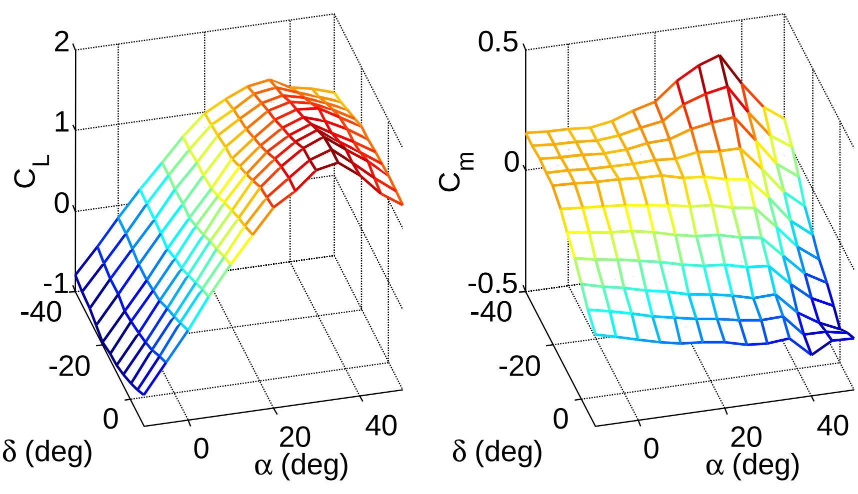
Angle of attack dynamics



$$\dot{\alpha} = -\frac{L(\alpha) - mg}{mV} + q$$

$$\dot{q} = \frac{M(\alpha, q, \delta)}{J}$$

Aerodynamic efforts



Ideas for control design

- Linearize the aircraft dynamics for a set of flight cases (**gain-scheduling**).
- Cancel the nonlinear system behaviour using feedback (**feedback linearization**).
- Refrain from cancelling "harmless" nonlinearities (**backstepping**).

2-stage control design

- Dynamics:

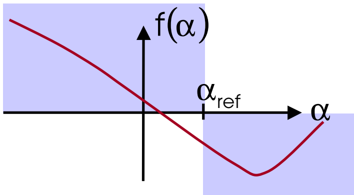
$$\dot{\alpha} = -\frac{L(\alpha) - mg}{mV} + q = f(\alpha) + q$$

$$\dot{q} = \frac{M(\alpha, q, \delta)}{J} = u$$

1. Design $u = k(\alpha, q)$
2. Allocate control surfaces.

Backstepping

- First consider $\dot{\alpha} = f(\alpha) + q$



- Virtual control law:

$$q_{des} = -f(\alpha_{ref}) - k_1(\alpha - \alpha_{ref})$$

Backstepping, contd.

- Create Lyapunov function

$$V = F(\alpha - \alpha_{ref}) + (q - q_{des})^2$$

- Demand

$$\frac{dV}{dt} = -W < 0$$

and solve for u.

Resulting control law

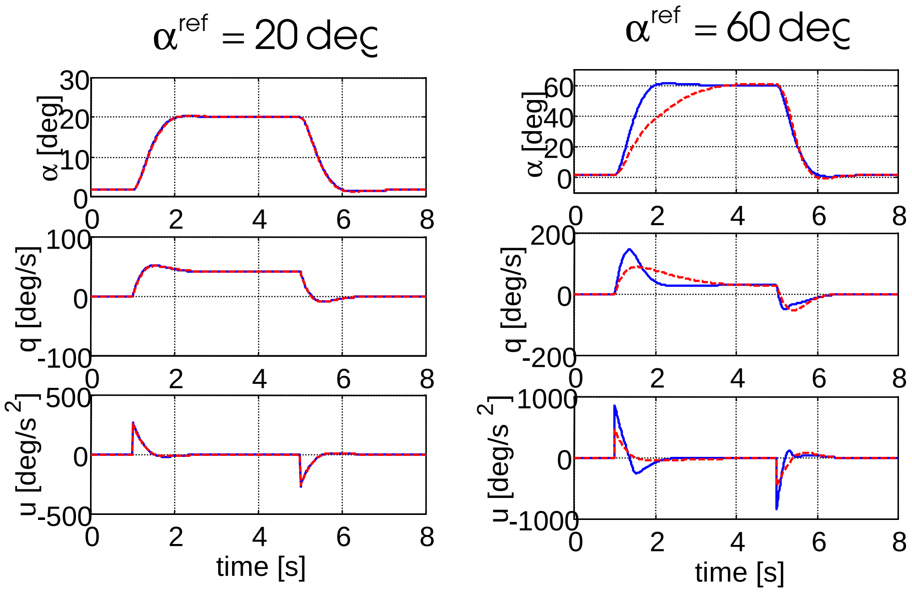
- Backstepping

$$u = -k_1(\alpha - \alpha_{ref}) - k_2(q + f(\alpha_{ref}))$$

- Feedback linearization

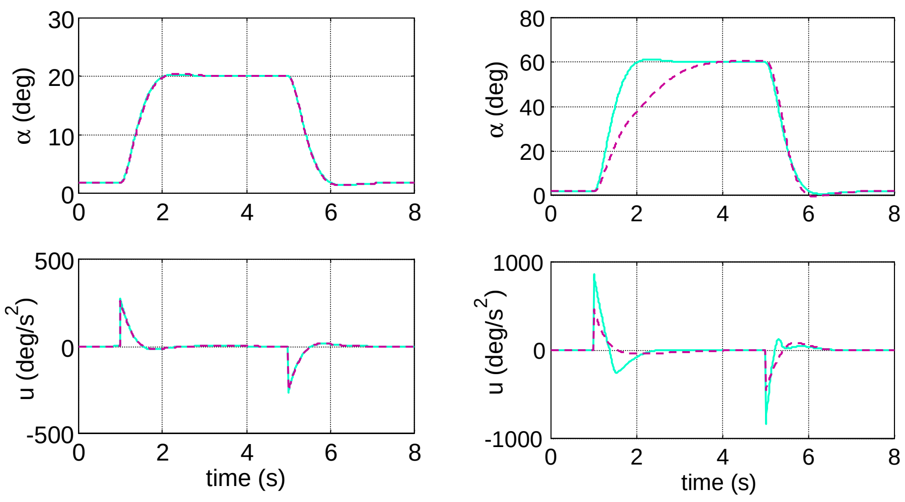
$$u = -k_1(\alpha - \alpha_{ref}) - (k_2 + f'(\alpha))(q + f(\alpha))$$

Step responses



Step responses

$\alpha^{ref} = 20$ deg $\qquad \alpha^{ref} = 60$ deg



Control law properties

$$\frac{M(\alpha, q, \delta)}{J} = -k_1(\alpha - \alpha_{ref}) - k_2(q + f(\alpha_{ref}))$$

- Global stability.
- Does not involve $dL/d\alpha$.
- Inverse optimal $\Rightarrow$ gain margin.

Full controller

$$\begin{aligned} \dot{\alpha} &= f_1(\alpha, \beta, p_s) + q \\ \dot{q} &= u_1 \end{aligned} \;\Bigg\}\Rightarrow u_1 = k_1\left(\alpha^{ref}, \alpha, q, \beta, p_s\right)$$

$$\begin{aligned} \dot{\beta} &= f_2(\beta, \alpha) + r_s \\ \dot{r}_s &= u_2 \end{aligned} \;\Bigg\}\Rightarrow u_2 = k_2(\beta, r_s)$$

$$\dot{p}_s = u_3 \;\}\Rightarrow u_3 = K\left(p_s^{ref} - p_s\right)$$

$$u = SJ^{-1}\left(M(\delta) - \omega \times J\omega\right)$$

Simulations