Nonlinear multivariable flight control

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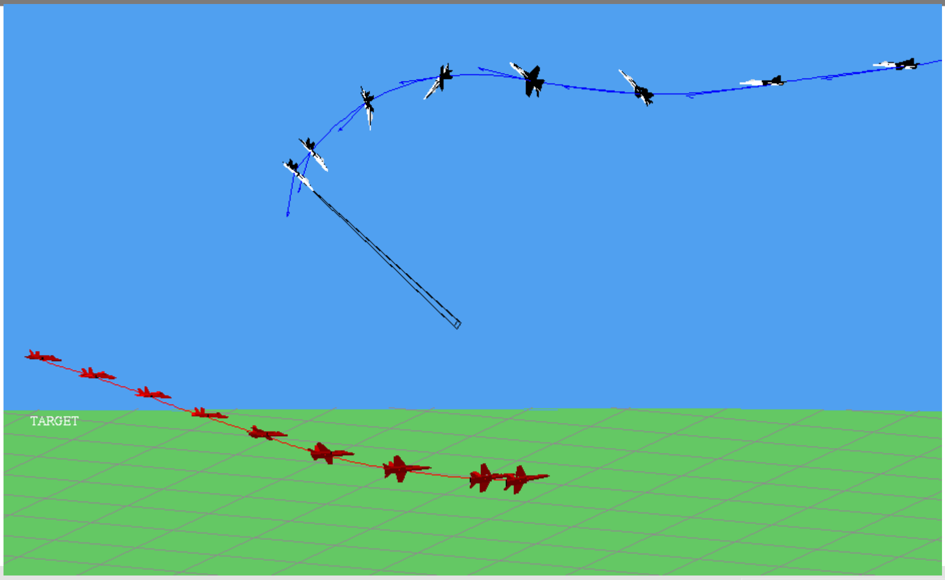
Background

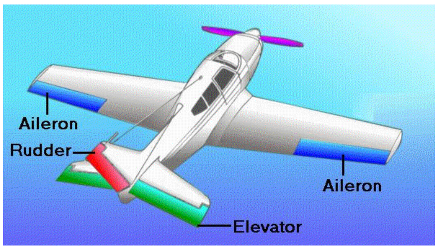
Nonlinear
- Different flight cases
- High angle-of-attack
- Rigid body dynamics

Multivariable
- 6 DOF
- Control surface redundancy
- Unconventional control surfaces

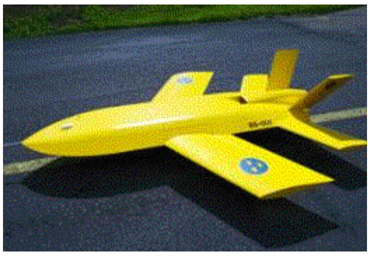
High angle of attack

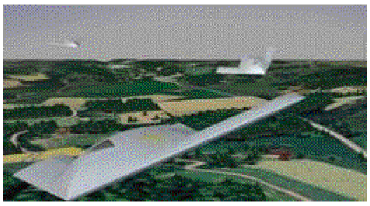


Unconventional control surfaces



vs.





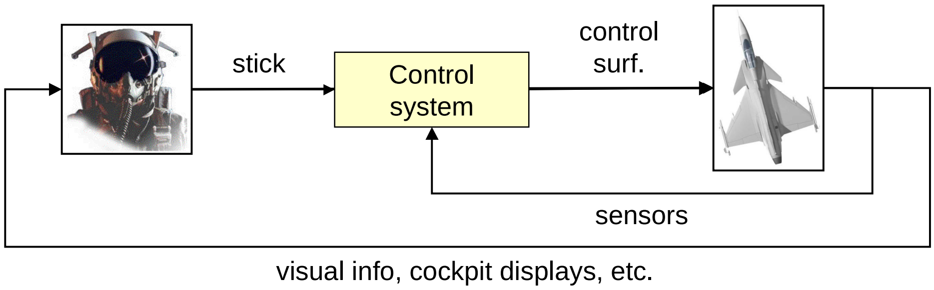
Outline

- Aircraft
- Backstepping
- Control allocation

Outline

- Aircraft
  - Why fly-by-wire?
  - Control objectives
  - Actuators
- Backstepping
- Control allocation

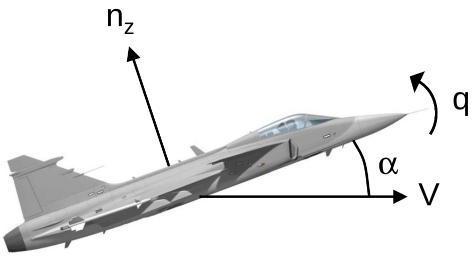
Why fly-by-wire?



- Stabilize aircraft
- Handling qualities
- Advanced control surfaces
- Autopilot functionality

Control objectives

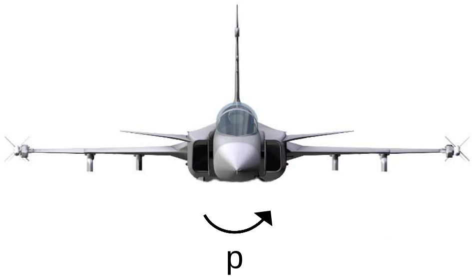
- Longitudinal control



  - Pitch rate
  - Load factor
  - Angle of attack

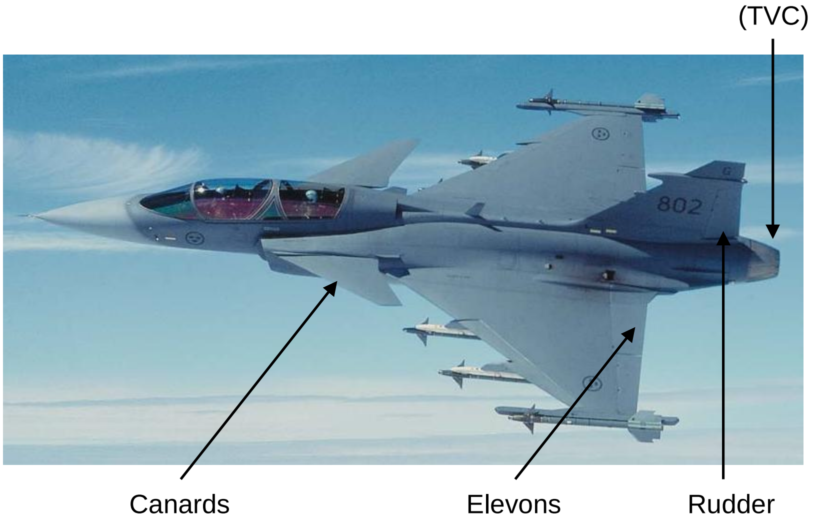
Control objectives

- Lateral control



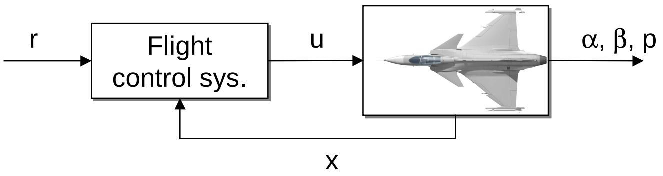
- Sideslip angle
- Roll rate

Actuators

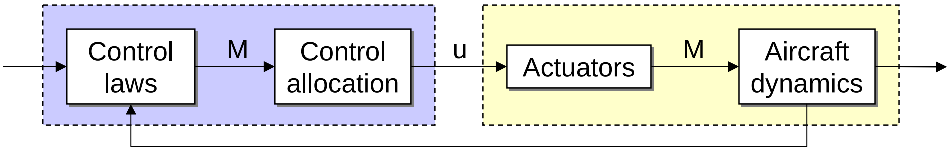


System overview

- Inner control loop:



- Modular control design:



Outline

- Aircraft
- Backstepping
  - What is backstepping?
  - Why use it?
  - Research at LiTH
- Control allocation

What is backstepping?

- **Constructive** nonlinear control design method.
- Model structure:

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2)\\ \dot{x}_2 &= f_2(x_1, x_2, x_3)\\ &\vdots\\ \dot{x}_n &= f_n(x_1, x_2, x_3, \ldots, x_n, u)\end{aligned}$$

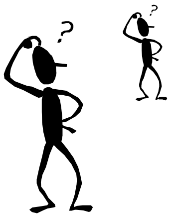
Same requirement as in feedback linearization

Why use backstepping?

- Can benefit from “useful” nonlinearities

  May require less
  - control effort
  - modeling information → robustness
- Can achieve GAS when feedback linearization fails

Design procedure



$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2)\\ \dot{x}_2 &= f_2(x_1, x_2, x_3)\\ &\vdots\\ \dot{x}_n &= f_n(x_1, x_2, x_3, \ldots, x_n, u)\end{aligned}$$

$V_1 = x_1^2$ decreases if

$x_2 = x_2^{des}(x_1)$

Step backwards

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2)\\ \dot{x}_2 &= f_2(x_1, x_2, x_3)\\ &\vdots\\ \dot{x}_n &= f_n(x_1, x_2, x_3, \ldots, x_n, u)\end{aligned}$$

$V_2 = V_1 + (x_2 - x_2^{des})^2$ decreases if

$x_3 = x_3^{des}(x_1, x_2)$

Step backwards

$$\begin{aligned}\dot{x}_1 &= f_1(x_1, x_2)\\ \dot{x}_2 &= f_2(x_1, x_2, x_3)\\ &\vdots\\ \dot{x}_n &= f_n(x_1, x_2, x_3, \ldots, x_n, u)\end{aligned}$$

$V_n = V_{n-1} + \left(x_n - x_n^{des}\right)^2$ decreases if

$$u = k(x_1, x_2, x_3, \ldots, x_n)$$

Research at LiTH

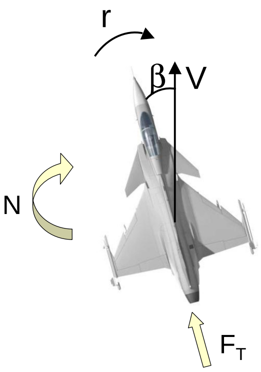
- Are there useful nonlinearities in the aircraft dynamics?

! Well, at least harmless.

- Can backstepping be applied to multivariable flight control?

! Yes, applicable to general rigid body dynamics.

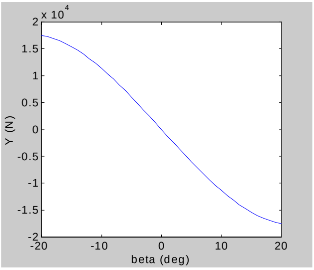
Sideslip regulation



$$\dot{\beta} = \frac{1}{mV}\left(Y(\beta) - F_T \sin\beta\right) - r$$

$$\dot{r} = \frac{1}{J_z} N(\beta, r, u)$$

Sideforce Y(β)



Backstepping ⟹ $\mathbf{N = -k_1\beta - k_2 r}$

- Linear
- Independent of Y(β)
- Inverse optimal

Outline

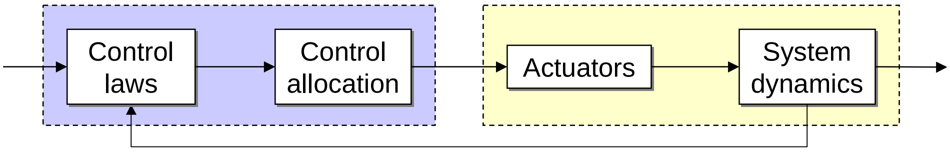
- Aircraft
- Backstepping
- Control allocation
  - What is control allocation?
  - Why use it?
  - Research at LiTH

What is control allocation?

How should the total control effort be distributed among the actuators?

Control design:

- Determine desired total control effort
- Distribute the control effort among the actuators



Control laws

Control allocation

Actuators

System dynamics

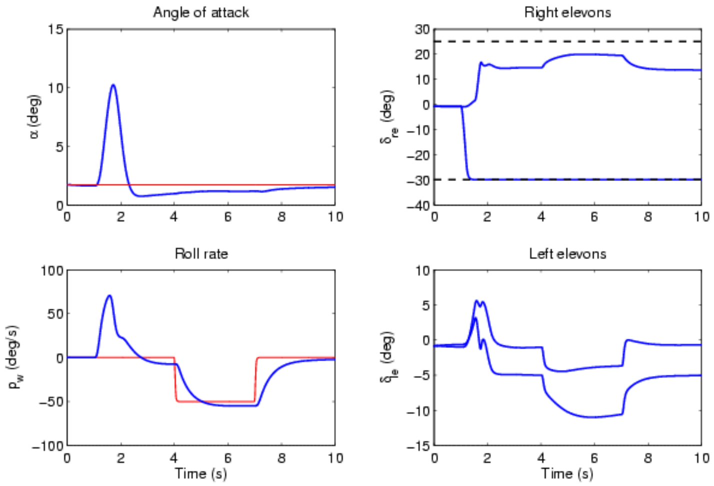
Applications

Why use control allocation?

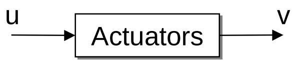
- Easy to reconfigure
- Cheap way to handle actuator limits
  - ”Poor man’s MPC”
- Necessary for certain control design methods
  - Feedback linearization (NDI)
  - Backstepping
- Modularity

Example: Hardover

Max deflection
after 1 s



Mathematical formulation



- u = true control signal
- v = virtual control signal (total control effort)
- Model: v = g(u)
- Linearization: v = Bu
- Constraints: $\left.\begin{array}{l} u_{min} \le u \le u_{max} \\ \dot{u}_{min} \le \dot{u} \le \dot{u}_{max} \end{array}\right\} \quad \underline{u}(t) \le u \le \bar{u}(t)$

Example

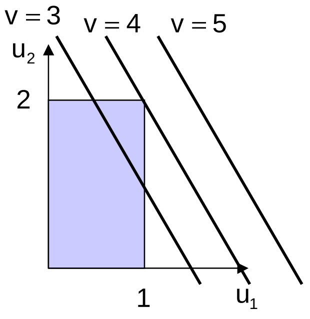
Dynamics: $\dot{x} = -2u_1 - u_2 \Leftrightarrow \begin{cases} \dot{x} = -v \\ v = 2u_1 + u_2 \end{cases}$

Constraints: $0 \le u_1 \le 1$

$0 \le u_2 \le 2$

Control law: $v = x$

Allocation problem: $2u_1 + u_2 = v$



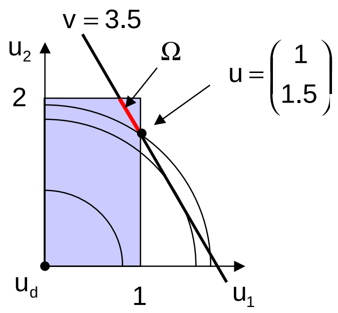
Optimization based control allocation

Minimize cost function.

$$\boxed{\begin{matrix} Bu = v \\ \underline{u} \le u \le \overline{u} \end{matrix}}$$

$$\boxed{\begin{matrix} 2u_1 + u_2 = v \\ 0 \le u_1 \le 1 \\ 0 \le u_2 \le 2 \end{matrix}} \quad \boxed{\begin{matrix} u_d = 0 \\ W_u = I \\ W_v = 1 \end{matrix}}$$

- $\Omega = \arg \min_{\underline{u} \le u \le \overline{u}} \|W_v(Bu - v)\|_2$
- $u = \arg \min_{u \in \Omega} \|W_u(u - u_d)\|_2$



Research at LiTH

- Can standard QP methods be used for control allocation in real time?
- ! Yes.

- How can filtering be included in the allocation?
- ! Also penalize *changes* in the control signal.

- How is control allocation related to LQ control?
- ! Equivalent without constraints.

Dynamic control allocation

- What?

$$v \longrightarrow \boxed{G(s)} \longrightarrow u \qquad \text{Constraints:} \quad \boxed{\begin{array}{c} Bu = v \\ \underline{u} \le u \le \overline{u} \end{array}}$$

- Why?
  - Actuator dynamics
  - ”Practical aspects”

Dynamic control allocation

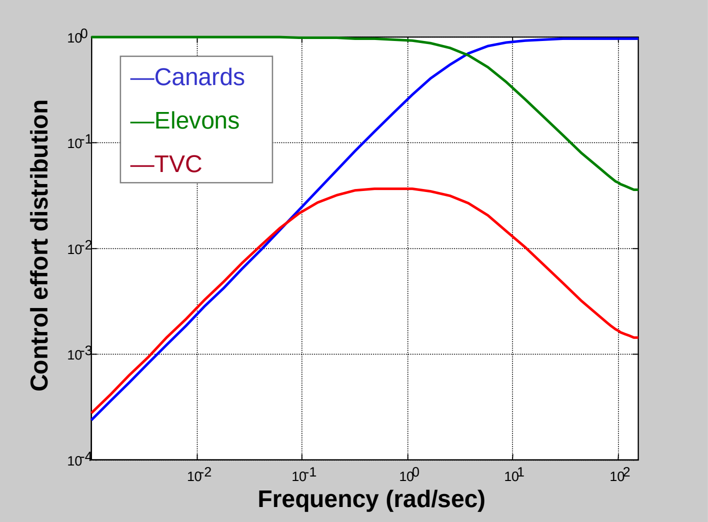
- How?

Also penalize changes in the control signal.

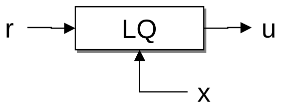
$$\begin{aligned} &\min_{u(t)} \|W_1 u(t)\|_2^2 + \|W_2 (u(t) - u(t-T))\|_2^2 \\ &Bu = v \end{aligned} \quad \Rightarrow \quad \boxed{u(t) = Fu(t-T) + Gv(t)}$$

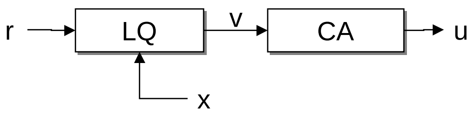
Stable linear filter

Example: Flight control



Control allocation vs LQ control





$\dot{x} = Ax + B_u u$

$\min_u \int_0^\infty x^T Q_1 x + u^T R_1 u \, dt$

$\boxed{u = -L_1 x}$

$B_u = B_v B$

$\Longleftrightarrow$

$\dot{x} = Ax + B_v v$

$v = Bu$

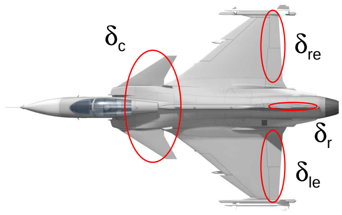
$\min_v \int_0^\infty x^T Q_2 x + v^T R_2 v \, dt$ $\quad \boxed{v = -L_2 x}$

$\min_u \|Wu\|_2$ då $Bu = v$ $\quad \boxed{u = L_3 v}$

$\boxed{u = -L_3 L_2 x}$

Q_1, R_1 $\Longleftrightarrow$ Q_2, R_2, W

Example

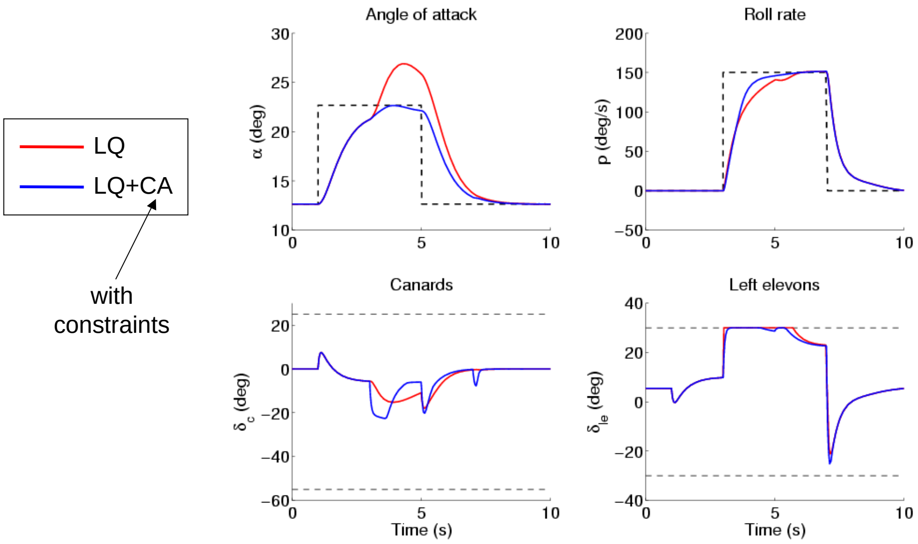


- Admire (FOI)
  - Mach 0.22, 3000 m
  - x = (α β p q r)
- Approximations:
  - Ignore actuator dynamics
  - View control surfaces as moment generators
- Model (for control): $\dot{x} = Ax + B_v v$

$$v = B\delta$$

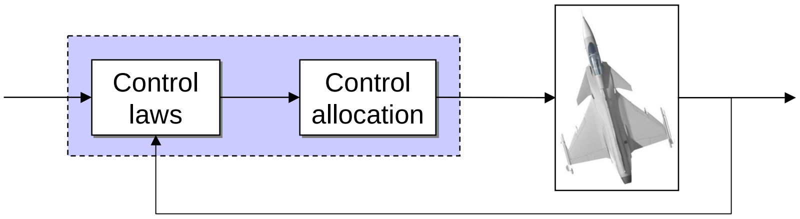
angular acc. control surfaces

Simulation results



with constraints

Summary



Configuration can handle

- Nonlinear dynamics
- Actuator redundancy
- Actuator constraints