Control Allocation

What, why, and how?

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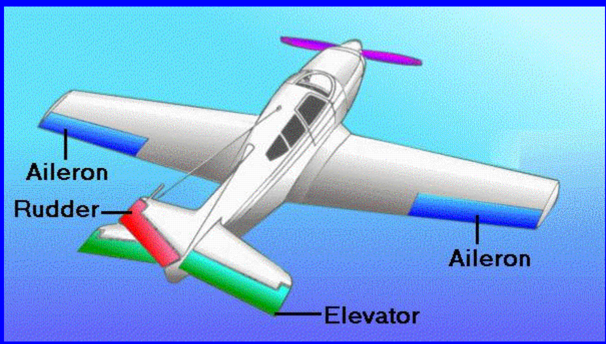
Aircraft maneuvering

The pilot controls

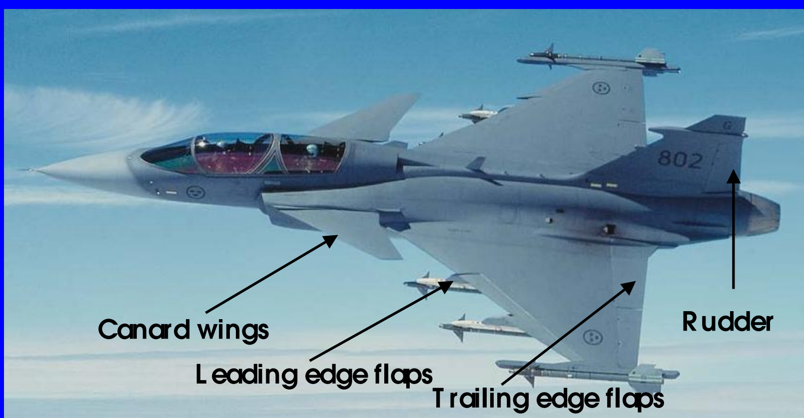
- Pitch
- Roll
- Yaw
- (Speed)

(Pitch, roll and yaw together: 3 DOF)

Traditional configuration



Modern configuration



Control allocation

? How do we distribute the control action among a redundant set of actuators?

Modular control design

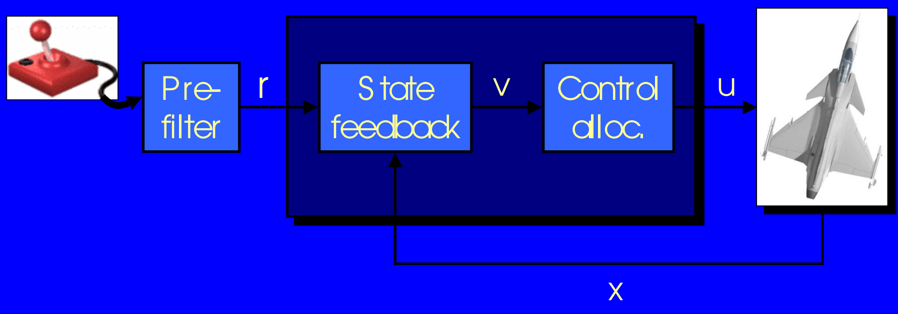
- Aircraft dynamics:

$$\dot{x} = f(x,u) \approx \tilde{f}(x, m(x,u)) = \tilde{f}(x,v)$$

dim u ≈ 10, dim v = 3

1. Design v=k(x,r) for closed loop performance.
2. Solve m(x,u) ≈ Bu=v for u.

Controller overview



Why is modularity good?

- Not all control designs methods handle redundancy.
- Separate control allocation simplifies actuator constraint handling.
- If an actuator fails, only control reallocation is needed.

Practical considerations

... while solving Bu=v:

- u is constrained in position and in rate.

  $\underline{u} \le u \le \bar{u}$

- The actuators have limited bandwidth.
- Actuators should not counteract eachother.
- Minimum-phase response.
- Want to minimize
  - drag
  - radar signature
  - structural load
- We are in a hurry! (50-100 Hz)

Solutions

$$Bu = v$$

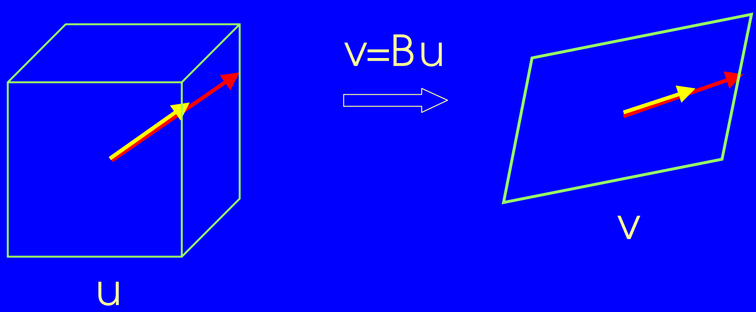
$$\underline{u} \le u \le \bar{u}$$

- Optimization based approaches
- Direct control allocation
- Daisy chaining

Daisy chaining

1. Use elevators until they saturate.
2. Use TVC for additional control.

Direct control allocation



Optimization based CA

$$\min_u f(u)$$
$$Bu = v$$
$$\underline{u} \leq u \leq \overline{u}$$

- How do we choose f(u)?
- Can we solve the problem in real time?

Pseudo-inverse

The optimal solution to

$$\min_u \|u\|_2$$
$$Bu = v$$

is given by $u = B^T(BB^T)^{-1}v = B^{\dagger}v$

Extension: $\min_u \|W(u - u_p)\|_2$

Half-time summary

So far, ***static*** CA: $u(t) = h(v(t))$

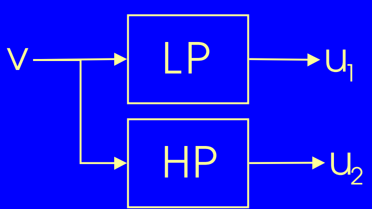
⇒

Same relative control distribution regardless of situation:

- maneuvering (transient)
- trimmed flight (steady state)

Dynamic control allocation

- Explicit filtering:



How can we impose

$$Bu = v$$
$$\underline{u} \le u \le \bar{u}$$

?

Incorporate filtering into an optimization framework.

Main idea

$$\min_{u(t)} \|W_1(u(t)-u_s(t))\|_2^2 + \|W_2(u(t)-u(t-1))\|_2^2$$
$$= \|W(u(t)-u_0(t))\|_2^2 + \ldots$$
$$Bu = v$$
$$\underline{u} \le u \le \overline{u}$$

- Stability?
- Control distribution?

The non-saturated case

$$\min_{u(t)} \|W_1(u(t)-u_s(t))\|_2^2 + \|W_2(u(t)-u(t-1))\|_2^2$$
$$Bu = v$$

is solved by

$$u(t) = Eu_s(t) + Fu(t-1) + Gv(t)$$

Is $v \rightarrow u$ stable?

<u>Thm:</u> If W_1 is non-singular then all eigenvalues of F satisfy

$$0 \le \lambda(F) < 1$$

- Asymptotically stable.
- Not oscillatory.

Steady state distribution?

<u>Thm:</u> If u_s satisfies

$$Bu_s = v$$

then

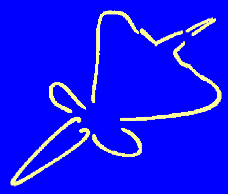
$$\lim_{t \to \infty} u(t) = u_s$$

u_s can be computed from

$$\min_{u_s} \left\| W(u_s - u_p) \right\|$$

$$Bu_s = v$$

Design example



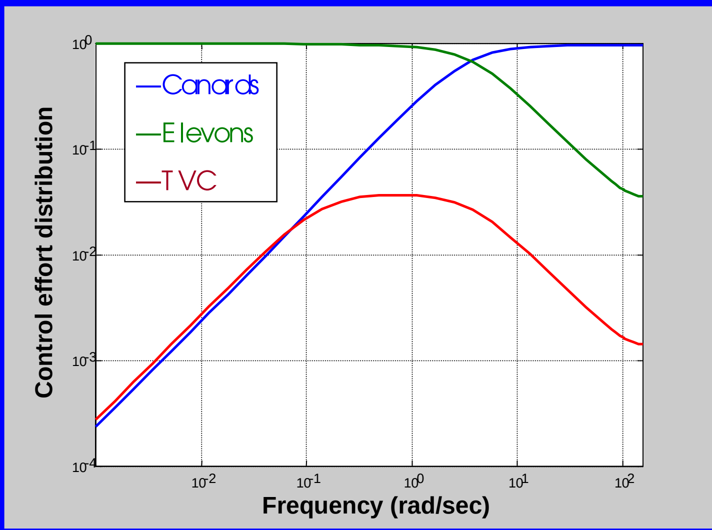
- Mach 0.5, 1000 m
- Pitching only
  - canards
  - elevons
  - TVC
- Trimmed flight: elevons only
- Canards: high frequencies
- TVC: midrange frequencies

Parameters

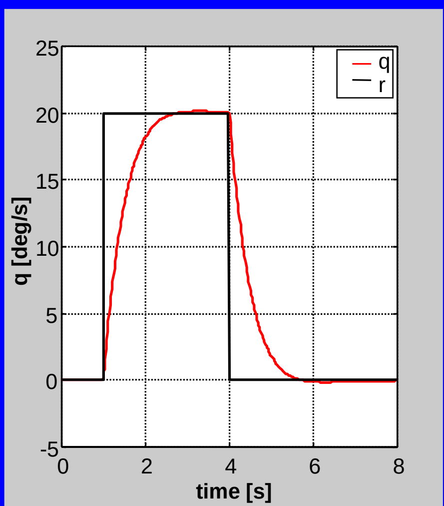
- Dynamics: $v = Bu = [8.0 \quad -20.2 \quad -0.87]u$
- Design variables:

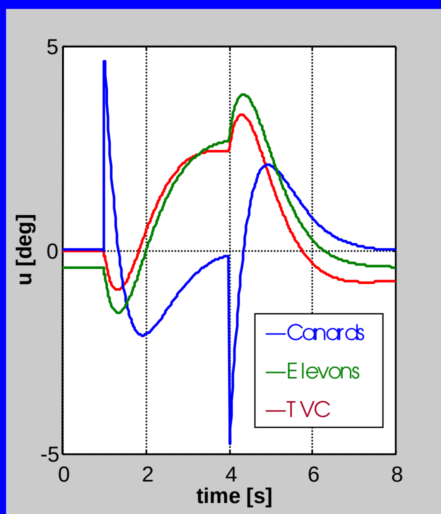
$$u_s = \begin{bmatrix} 0 \\ -1/20.2 \\ 0 \end{bmatrix} v \quad W_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \quad W_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Frequency distribution



Aircraft response





Computing the solution

$$\min_{u} \|W(u-u_0)\|_2$$
$$u \in \arg\min \|W_a(Bu-v)\|_2$$
$$\underline{u} \le u \le \overline{u}$$

Can this problem be solved in real-time?

Not according to the litterature.

Problem specific info

- Simple inequality bounds.
- From t-1:
  - u
  - active constraints
- Convergence in one sample not necessary.

Trim existing methods!

Summary

- Dynamic control allocation - a new concept.
- Need for efficient solvers.
- New field $\Rightarrow$ lot's to do!