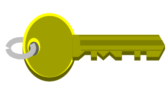
Dynamic Control Allocation using Constrained QP

Ola Härkegård
Linköping University
Sweden

How can we utilize actuator redundancy?



- Actuator limits
- Frequency division

Control allocation

$M = Bu$

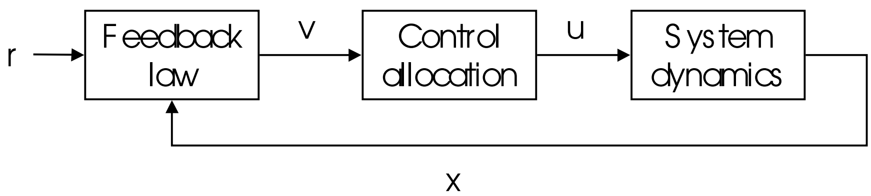
$\underline{u} \le u \le \bar{u}$

$F = Bu$

$\underline{u} \le u \le \bar{u}$

? Which u should we pick?

System overview



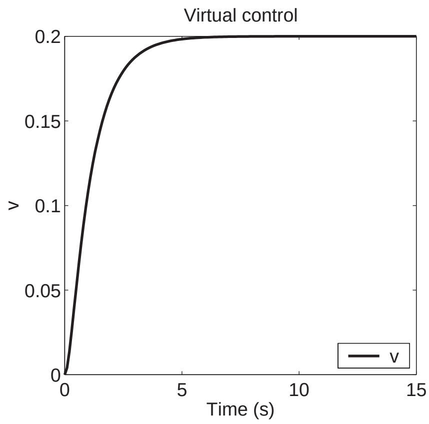
$v = Bu$

$\underline{u} \le u \le \bar{u}$

- Direct CA
- Daisy chaining
- Linear prog.
- Quadratic prog.

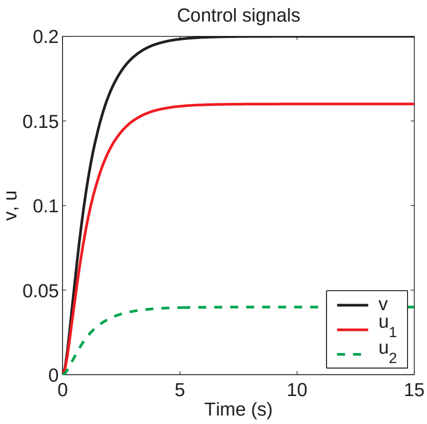
Example: $v = u_1 + u_2$

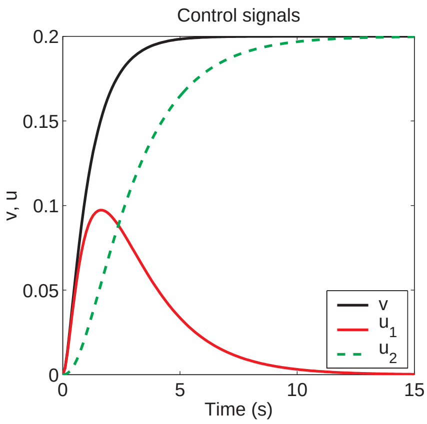
- Desired total control



Example: $v = u_1 + u_2$

- $\min \left\| \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \right\|_2$
- Dynamic allocation





Why dynamic allocation?

- Specify frequency range for each actuator.
- Improve closed loop behaviour.

How?

- Solve

$$\min_{u(t)} \| W_1(u(t) - u_s(t)) \|_2^2 + \| W_2(u(t) - u(t-T)) \|_2^2$$

$$Bu(t) = v(t)$$

$$\underline{u}(t) \leq u(t) \leq \overline{u}(t)$$

- W_1, W_2 → frequency characteristics
- u_s → steady state distribution

Properties

- Without actuator constraints:

$$\min_{u(t)} \| W_1(u(t) - u_s(t)) \|_2^2 + \| W_2(u(t) - u(t-T)) \|_2^2$$
$$Bu(t) = v(t)$$

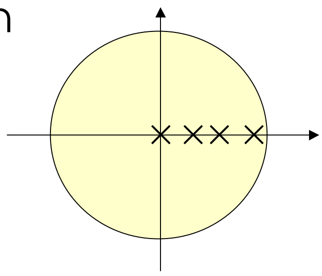
- Solution: $u(t) = Eu_s(t) + Fu(t-T) + Gv(t)$

Stability

- Filter: $u(t) = Eu_s(t) + Fu(t-T) + Gv(t)$
- If W_1 is nonsingular then

$$0 \le \lambda(F) < 1$$

(asymptotically stable)



Steady state

$$\min_{u(t)} \| W_1(u(t) - u_s(t)) \|_2^2 + \underbrace{\| W_2(u(t) - u(t-T)) \|_2^2}_{=0}$$

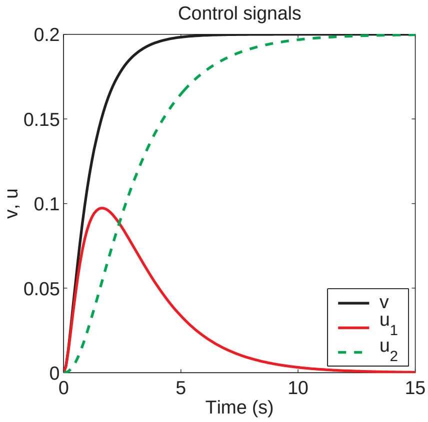
$$Bu(t) = v(t)$$

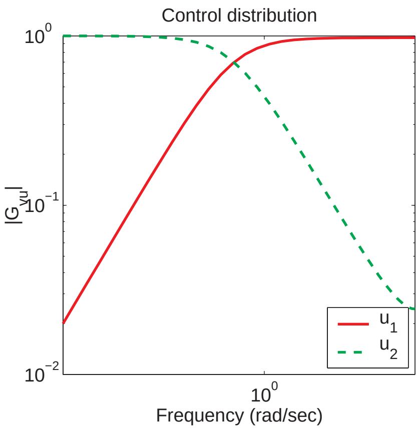
- If $Bu_s(t)=v(t)$ then

$$u(t) \to u_s(t) \quad \text{as} \quad t \to \infty$$

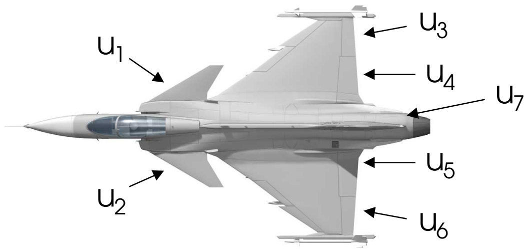
Example: $v=u_1+u_2$

$$u_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v \quad W_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad W_2 = \begin{pmatrix} 0 & 0 \\ 0 & 10 \end{pmatrix}$$





Admire



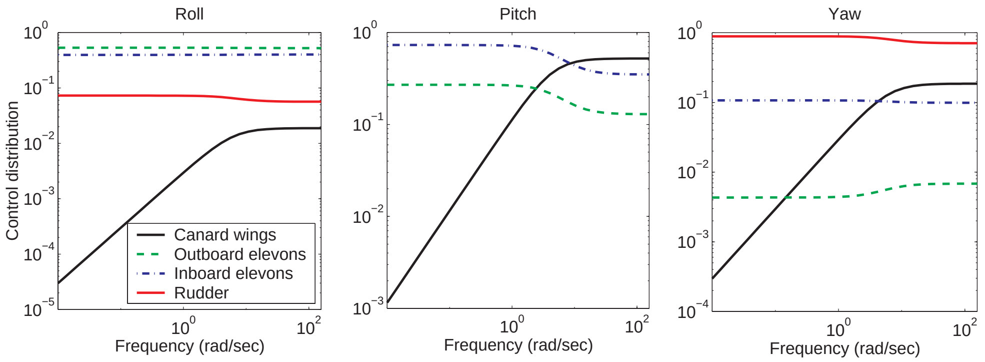
- $u = \begin{pmatrix} u_1 \\ \vdots \\ u_7 \end{pmatrix}$ $v = \begin{pmatrix} C_l \\ C_m \\ C_n \end{pmatrix}$ $B = \begin{pmatrix} & 3 \times 7 & \end{pmatrix}$
- 1000 m, Mach 0.5
- Canards for HF
  - Minimum drag
  - Improved n_z response

Design parameters

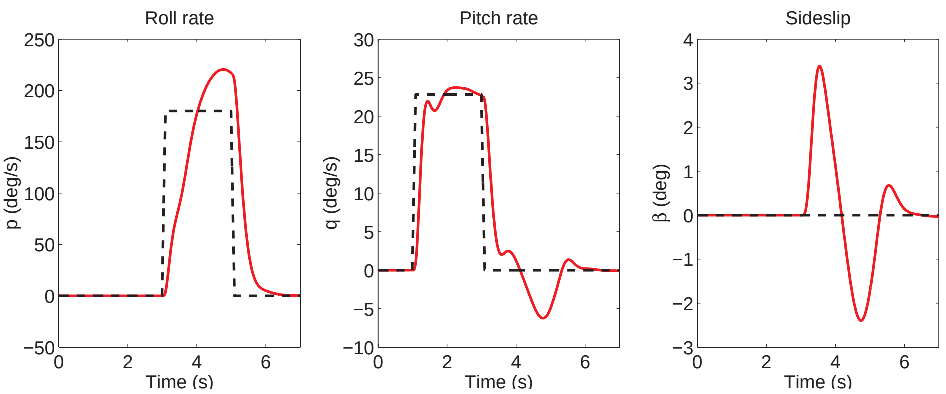
- $$\begin{aligned} u_s &= \arg\min \|u\|_2 \\ Bu &= v \\ u_1 = u_2 &= 0 \end{aligned} \Rightarrow u_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \\ & \vdots & \end{pmatrix} v$$
- $W_1 = \mathrm{diag}(2 \quad \ldots \quad 2)$, $W_2 = \mathrm{diag}(5 \quad 5 \quad 10 \quad \ldots \quad 10)$

$$u(t) = Fu(t - T) + Gv(t)$$

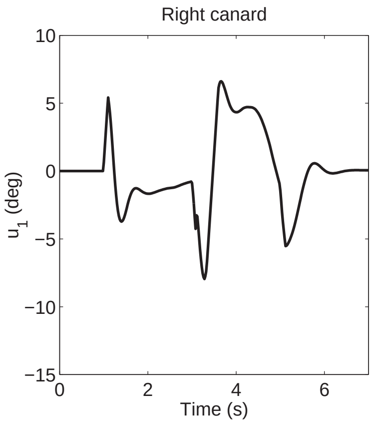
Control distribution

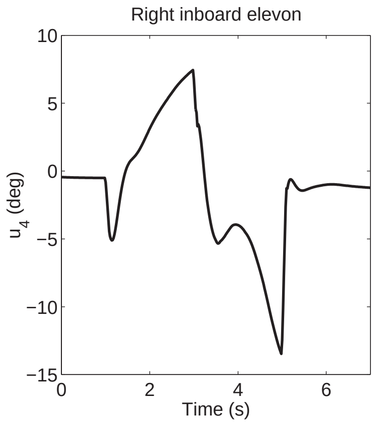


Simulation results



Control surfaces





Dynamic vs static