Vector backstepping design for flight control

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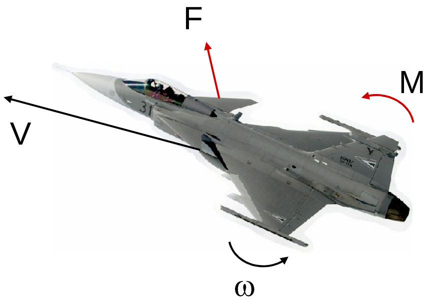
Rigid body control problem

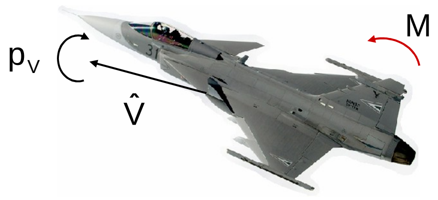
- States:
  - V = velocity
  - ω = angular velocity
- Inputs:
  - F = force (aero, engine, gravity)
  - M = tourque (aero, control surf)



Control objective

- Controlled variables:
  - α = angle of attack
  - β = angle of sideslip
  - p_V = velocity vector roll rate
- Control input:
  - M = torque
- Relationship:

$$\hat{V} = (\cos\alpha\cos\beta \quad \sin\beta \quad \sin\alpha\cos\beta)^T$$
$$\omega^T\hat{V} = p_V$$



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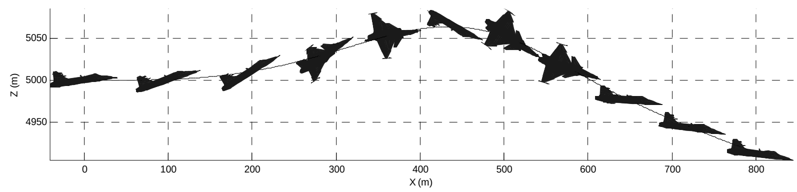
Example



$\alpha_{ref} = 25$ deg

$\beta_{ref} = 0$ deg

$p_{V,ref} = 60$ deg/ s

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Main idea

Consider **rigid body dynamics** on **vector form:**

$$m\dot{V} = -\omega \times mV + F$$
$$J\dot{\omega} = -\omega \times J\omega + M$$

Use **backstepping** to design control law $M(V,\omega)$ to achieve

$$\hat{V} = \hat{V}_{ref}(\alpha_{ref}, \beta_{ref})$$
$$\omega^T \hat{V} = p_{V,ref}$$

Related backstepping papers

- Spacecraft
  - Krstić and Tsiotras, "*Inverse optimal stabilization of a rigid spacecraft*"
- Helicopters
  - Hamel and Mahony, *"Visual servoing of an under-actuated dynamic rigid-body system"*
- Marine vessels
  - Fossen and Berge, *"Nonlinear vectorial backstepping design for global exponential tracking of marine vessels in the presence of actuator dynamics"*
- Mobile robots
  - Jiang and Nijmeijer, *"Tracking control of mobile robots: A case study in backstepping"*
- **Differences**:
  - Control objective
  - External forces
  - 3D vs 2D

Related aircraft control papers

- Feedback linearization
  - Lane and Stengel, *"Flight control design using non-linear inverse dynamics"*
  - Enns et al., *"Dynamic inversion: An evolving methodology for flight control design"*
- ...with time-scale separation
  - Reiner et al., *"Flight control design using robust dynamic inversion and time-scale separation"*

- **Differences:**
  - Design method
  - Component form vs vector form

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Aircraft dynamics

$$m\dot{V} = -\omega \times mV + F$$
$$J\dot{\omega} = -\omega \times J\omega + M$$

Introduce $V = |V|\hat{V}$

$$\dot{V}_T = \ldots \quad \Leftrightarrow \quad \tfrac{d}{dt}|V| = \tfrac{1}{m}\hat{V}^T F$$

$$\dot{\hat{V}} = \left[-\omega + \tfrac{1}{m|V|}\hat{V} \times F\right] \times \hat{V}$$
$$J\dot{\omega} = -\omega \times J\omega + M$$
$$\Leftrightarrow \quad \dot{\alpha} = \ldots,\ \dot{\beta} = \ldots,\ \dot{p} = \ldots,\ \dot{q} = \ldots,\ \dot{r} = \ldots$$

Assumption: $F = F(V,t)$ (independent of angular velocity and control surfaces)

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Stationary motion

Dynamics:

$$\dot{\hat{V}} = \left[-\omega + \tfrac{1}{m|V|}\hat{V} \times F\right] \times \hat{V}$$
$$J\dot{\omega} = -\omega \times J\omega + M$$

Consider a motion with $\hat{V} = \hat{V}_{ref}$ $(\alpha = \alpha_{ref}, \beta = \beta_{ref})$.

This requires

$$\omega = \tfrac{1}{m|V|}\hat{V}_0 \times F + \lambda \hat{V}_0$$

Arbitrary scalar representing **velocity vector roll rate.**

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Backstepping design (step 1)

- Dynamics: $\dot{\hat{V}} = \left[-\omega + \tfrac{1}{m|V|}\hat{V} \times F\right] \times \hat{V}$
- Control input: ω
- Lyapunov function: $W_1 = \tfrac{1}{2}\left\|\hat{V} - \hat{V}_{ref}\right\|^2$
- The control law $\omega_d = \tfrac{1}{m|V|}\hat{V} \times F(V,t) + p_{V,ref}\hat{V} - K_1(\hat{V} \times \hat{V}_0)$

achieves $\dot{W}_1\Big|_{\omega=\omega_d} = -\left\|\hat{V} \times \hat{V}_{ref}\right\|_{K_1}^2 < 0, \quad \hat{V} \neq \pm\hat{V}_{ref}$

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Backstepping design (step 2)

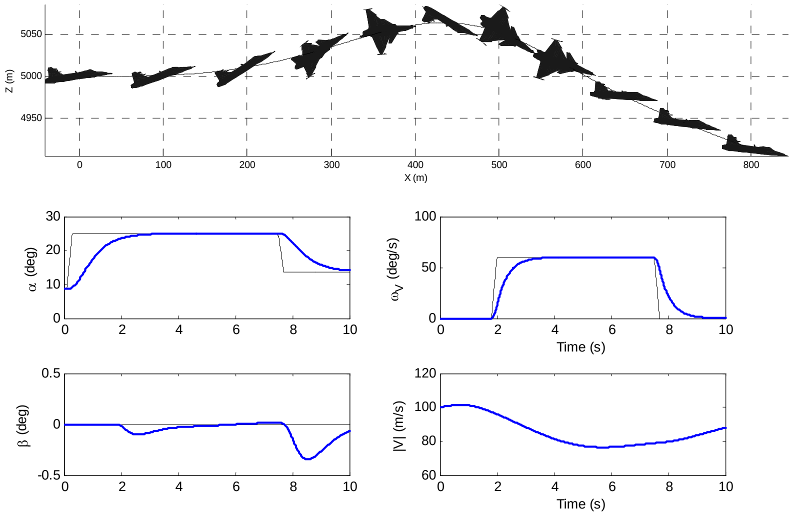
- Dynamics:
$$\dot{\hat{V}} = \left[-\tilde{\omega} + K_1\left(\hat{V} \times \hat{V}_{ref}\right)\right] \times \hat{V}$$
$$J\dot{\tilde{\omega}} = -J\dot{\omega}_d - \omega \times J\omega + M$$
where $\tilde{\omega} = \omega - \omega_d$
- Control input: M
- Lyapunov function: $W = \frac{c}{2}\left\|\hat{V} - \hat{V}_{ref}\right\|^2 + \left\|\tilde{\omega}\right\|_J^2$
- The control law
$$M = J\dot{\omega}_d + \omega \times J\omega - K_2\tilde{\omega}$$

achieves $\dot{W} < 0$ except at $\hat{V} = \pm\hat{V}_{ref},\ \omega = \omega_d$

Closed loop system

- Closed loop dynamics:
$$\dot{\hat{V}} = \left[-\tilde{\omega} + K_1\left(\hat{V} \times \hat{V}_{ref}\right)\right] \times \hat{V}$$
$$J\dot{\tilde{\omega}} = -K_2\tilde{\omega}$$
- Independent of F
- Nonlinear: $\beta = \beta_{ref} = 0$ gives (with particular choice of K_1 and K_2) :
$$\dot{\alpha} = -k_\alpha \sin\left(\alpha - \alpha_{ref}\right) + \tilde{q}$$
$$\dot{\tilde{q}} = -k_q\tilde{q}$$
- Linearization has real poles (limitation)

Example (Admire-like model, 5000 m, Mach 0.3)



Summary

- Nonlinear flight control design using **backstepping**
- Based on **vector form** description of dynamics
- Feedback gains from **linear closed loop requirements**