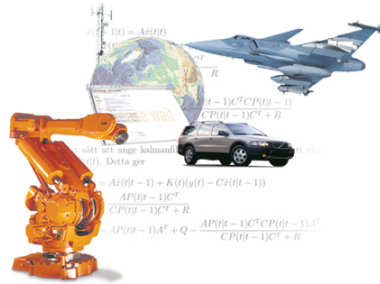


Efficient Active Set Algorithms for Solving Constrained LS Problems in Aircraft Control Allocation



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Main Message

Today: Efficient but approximate methods

? Can standard QP methods be used efficiently?

! Yes, complexity \approx pseudoinverse methods

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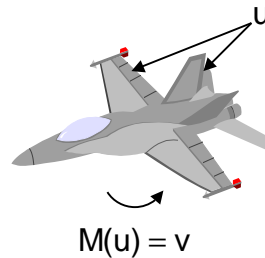


What is Control Allocation?

- Control design $\rightarrow M(u) = Bu = v$

- Actuator constraints

- Position: $u_{\min} \leq u \leq u_{\max}$
- Rate: $r_{\min} \leq \dot{u} \leq r_{\max}$



$$\dot{u}(t) \approx \frac{u(t) - u(t-T)}{T} \rightarrow$$

$$\begin{aligned} Bu(t) &= v(t) \\ \underline{u}(t) &\leq u(t) \leq \bar{u}(t) \end{aligned}$$

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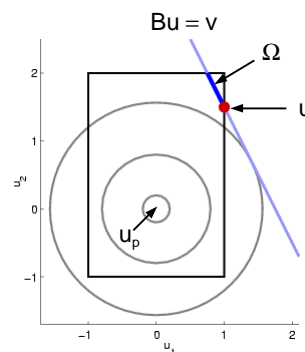


Least Squares Formulation

- Sequential least squares:

$$\Omega = \arg \min_{\underline{u} \leq u \leq \bar{u}} \|W_v(Bu - v)\|_2$$

$$\begin{aligned} u &= \arg \min_{u \in \Omega} \|W_u(u - u_p)\|_2 \\ u &\in \Omega \end{aligned}$$



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Numerical Methods

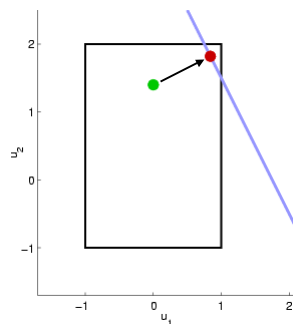
- Active set methods (this paper)
- Pseudoinverse methods (dominate)
- ...

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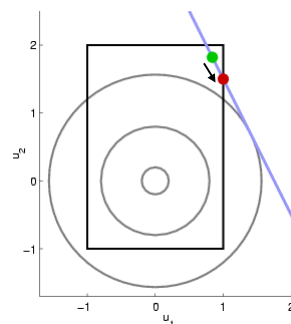
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Active Set

$$\textcircled{1} \min \|Bu - v\|_2$$
$$\underline{u} \leq u \leq \bar{u}$$



$$\textcircled{2} \min \|u\|_2$$
$$Bu = v$$
$$\underline{u} \leq u \leq \bar{u}$$

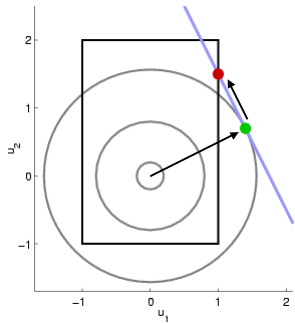


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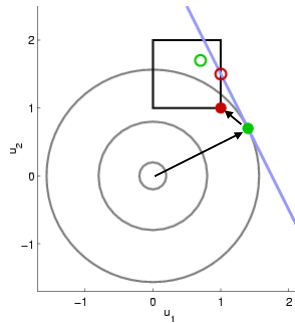
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Pseudoinverse

- Successful case



- Failing case



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Why Active Set?

- Always finds optimal solution
- Can reuse previous solution
- All iterates are feasible

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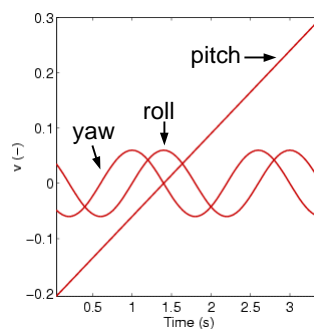


Example (Durham and Bordignon, 1996)



- 8 actuators, 3 moments
- Position and rate limits

▪ Aerodynamic coefficients



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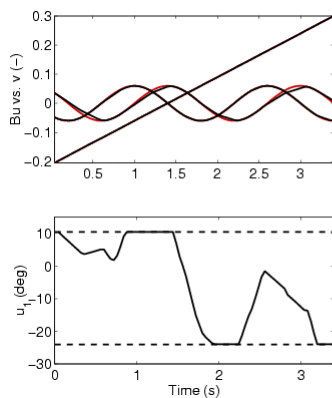
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Simulations

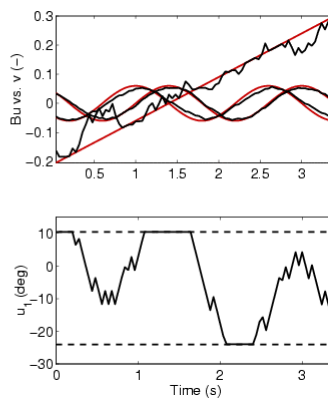
Active set

Mean: 0.9 ms Max: 2.5 ms



Pseudoinverse

Mean: 0.9 ms Max: 1.5 ms



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Conclusions

Active set methods are well suited for control allocation

- Find optimal control input
- Complexity \approx pseudoinverse methods