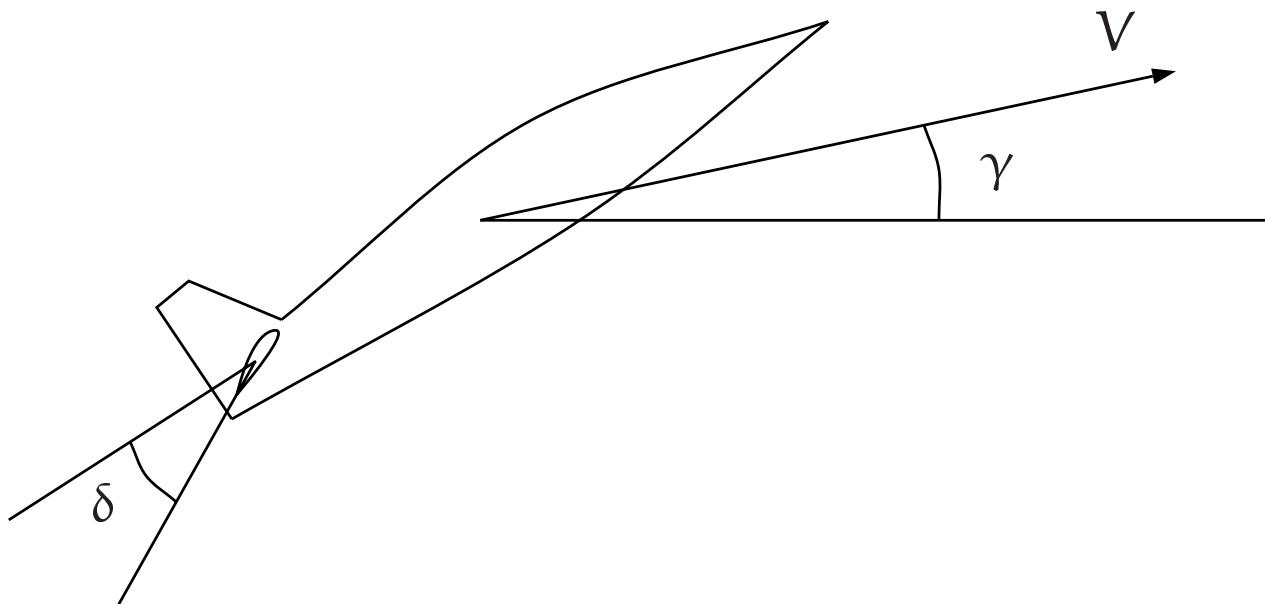


# A Backstepping design for Flight Path Angle Control

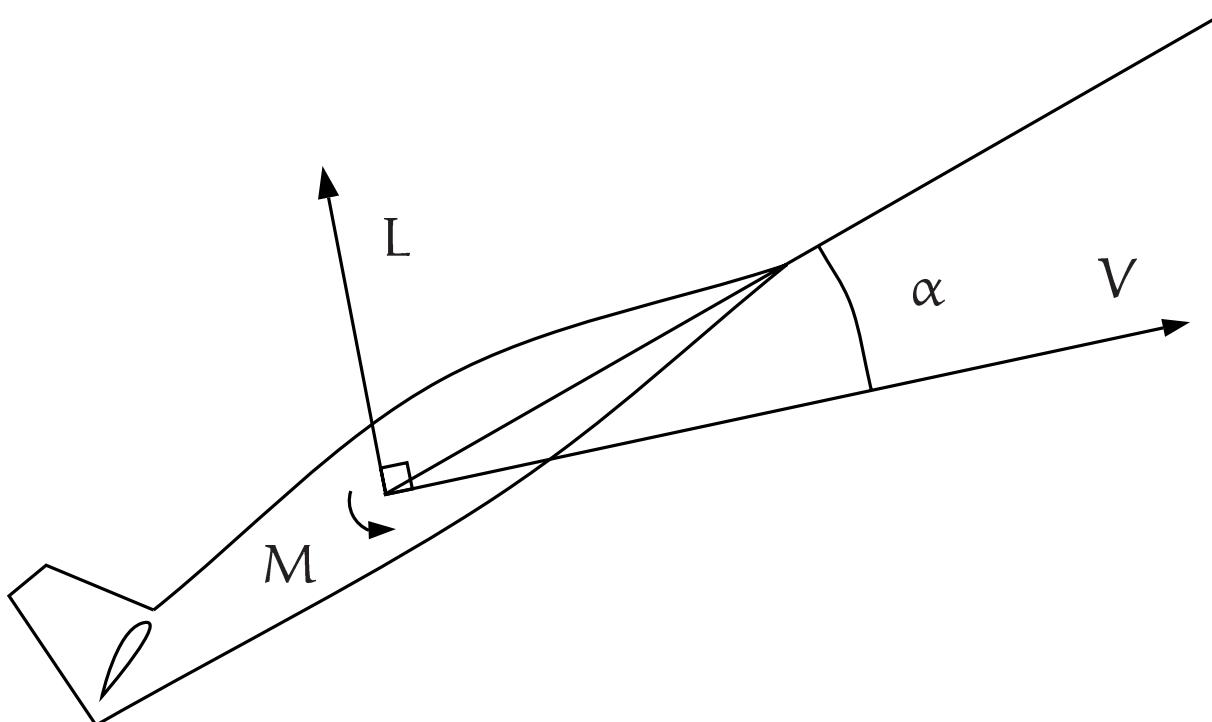
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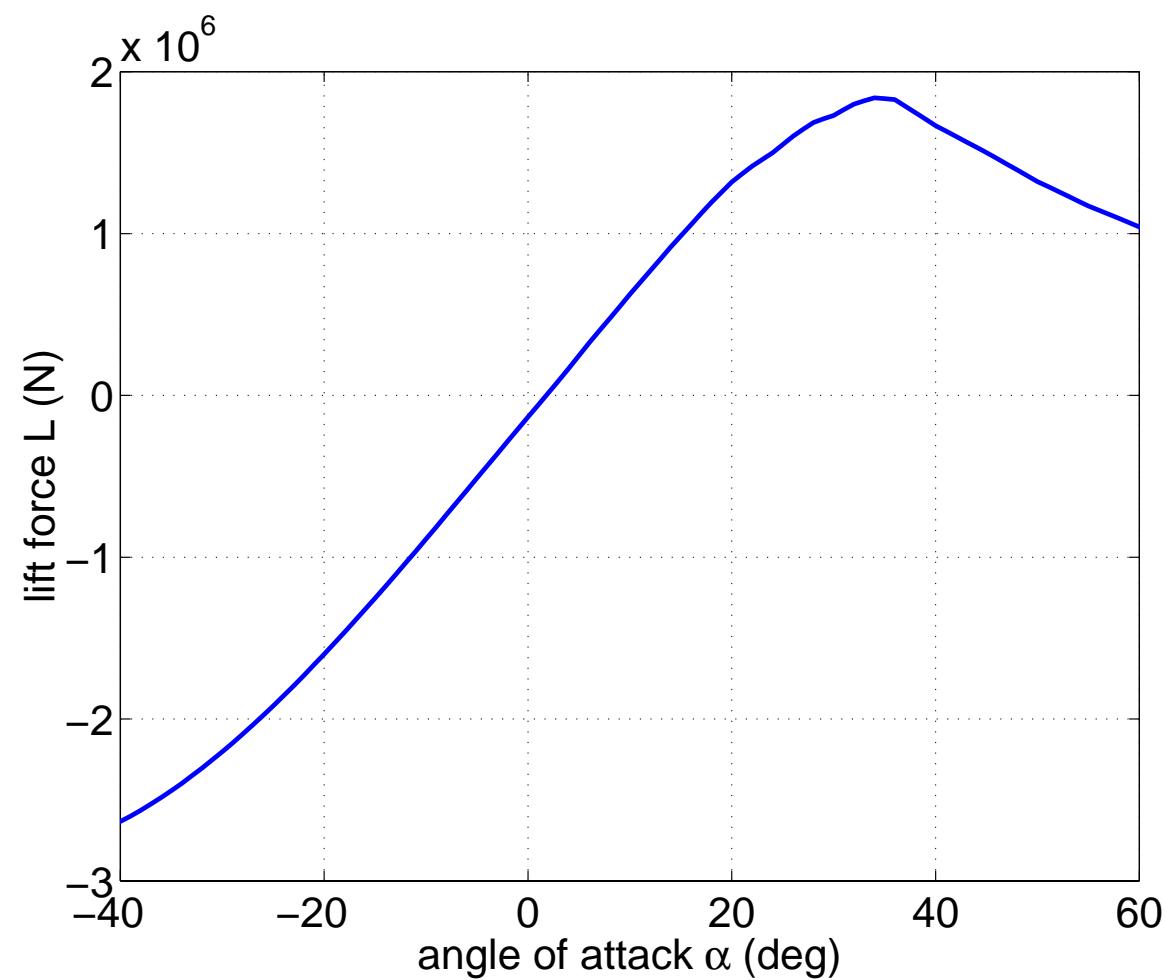
# Control Problem



# Nonlinear Aerodynamics



## Lift Force



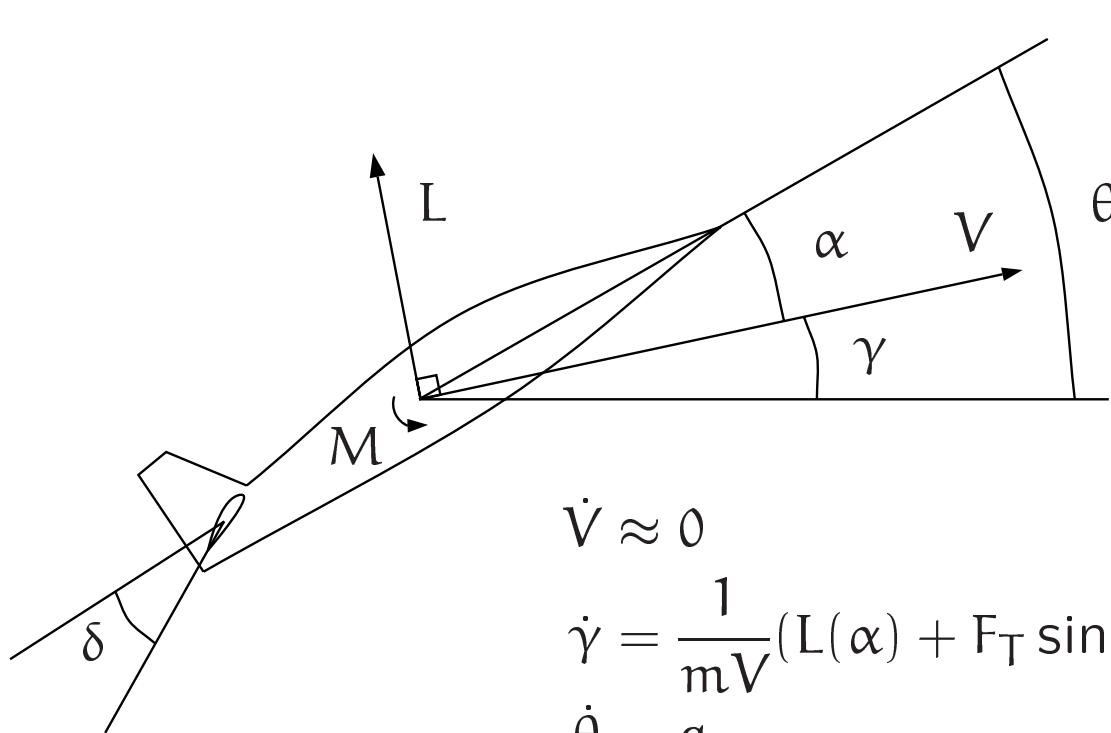
## Backstepping

$$\dot{x}_1 = f_1(x_1, \textcolor{red}{x}_2)$$

$$\dot{x}_2 = f_2(x_1, x_2, \textcolor{red}{x}_3)$$

$$\dot{x}_3 = f_3(x_1, x_2, x_3, \textcolor{red}{u})$$

## Aircraft Model



$$\dot{V} \approx 0$$

$$\dot{\gamma} = \frac{1}{mV}(L(\alpha) + F_T \sin \alpha - mg \cos \gamma)$$

$$\dot{\theta} = q$$

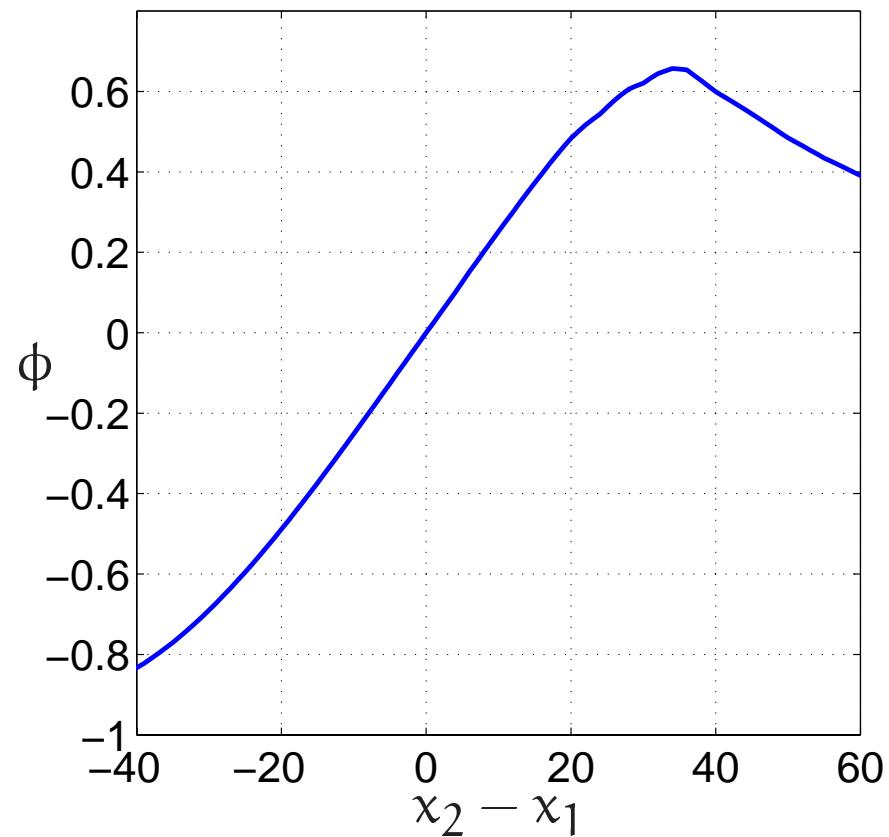
$$\dot{q} = u = \frac{1}{I_y}(M(\alpha, q, \delta) + F_T Z_{TP})$$

## Core Problem

$$\dot{x}_1 = \phi(x_2 - x_1)$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = u$$



## Backstepping Design

1.

$$V_1 = \frac{1}{2}x_1^2$$

$$\dot{V}_1 = x_1 \phi(x_2 - x_1) < 0 \quad \text{for } x_2^{\text{des}} = -k_1 x_1$$

2.

$$V_2 = c_1 V_1 + F(x_1) + \frac{1}{2}(x_2 - x_2^{\text{des}})^2$$

$$\dot{V}_2 < 0 \quad \text{for } x_3^{\text{des}} = -k_2(x_2 - x_2^{\text{des}})$$

3.

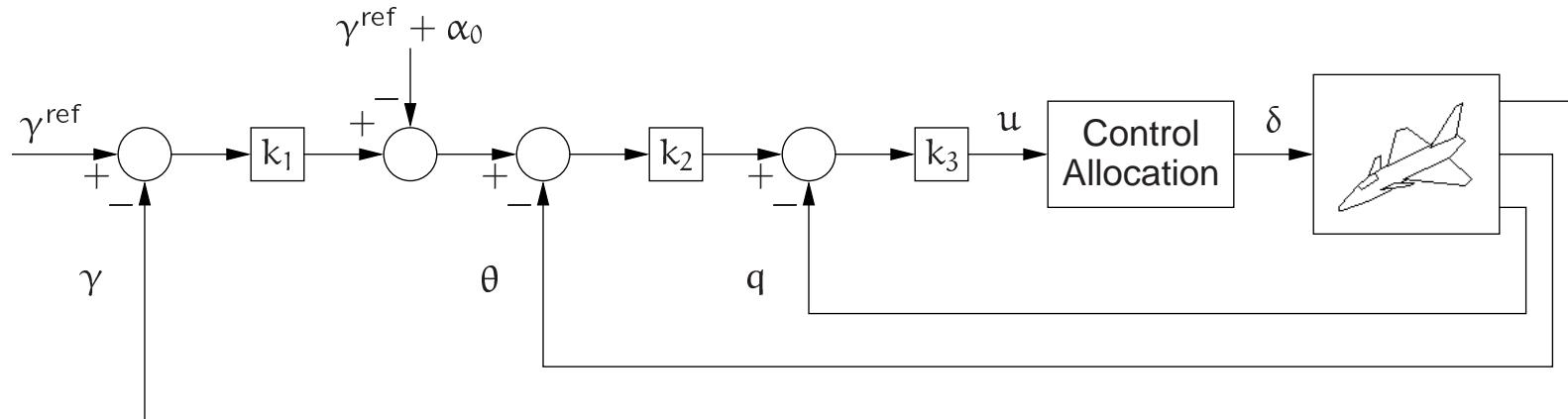
$$V_3 = c_2 V_2 + \frac{1}{2} (x_3 - x_3^{\text{des}})^2$$
$$\dot{V}_3 < 0$$

if we select

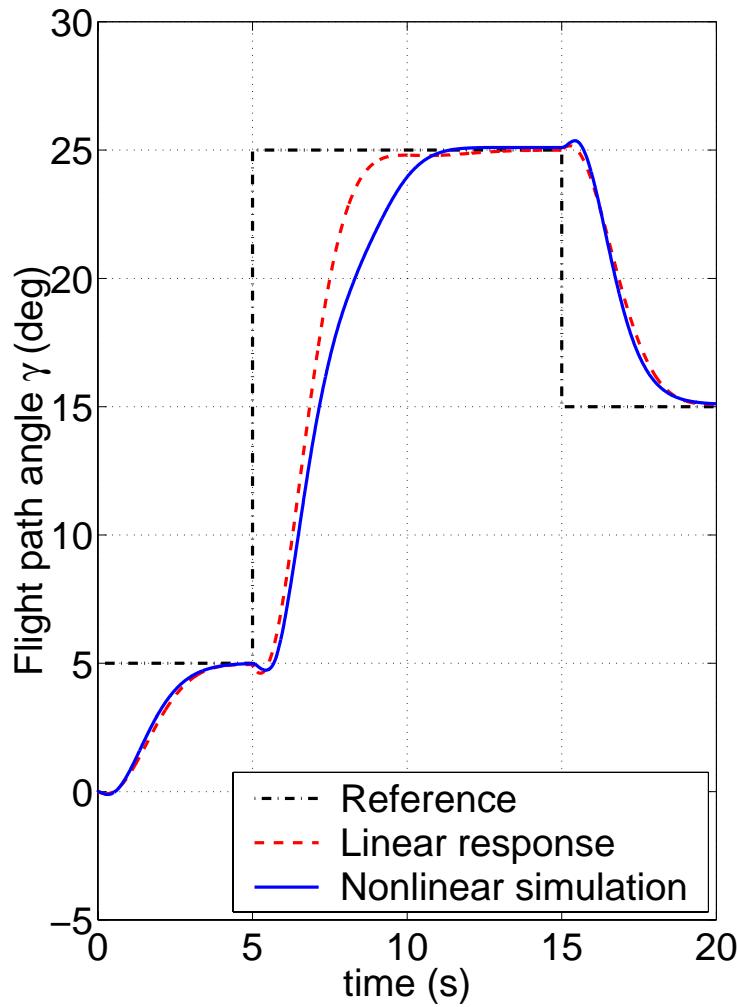
$$u = -k_3(x_3 - x_3^{\text{des}}) = -k_3(x_3 + k_2(x_2 + k_1x_1))$$

*This linear control law globally stabilizes the nonlinear system.*

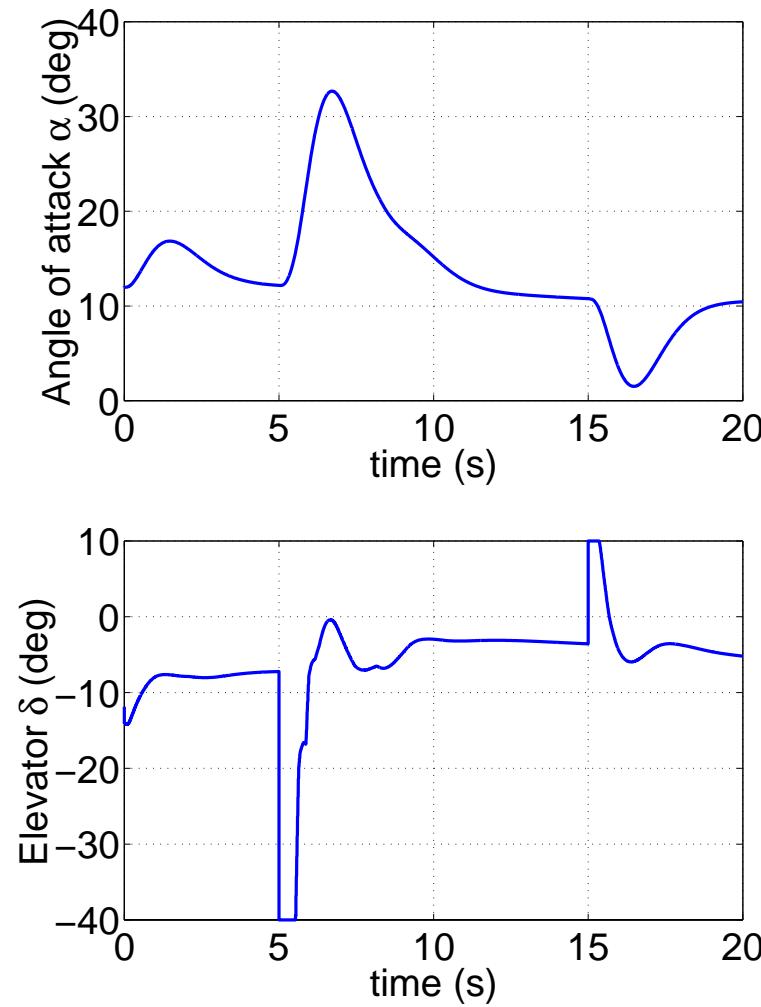
## Closing the Loop



Design parameters:  $k_1 > -1, k_3 > k_2 > 0$



## Simulations



## Key Properties of the Control Law

- Linear in the moment
- Requires little knowledge of the lift
- Globally stabilizing
- Gain margin

## Current Research

- Angle of attack and sideslip control
- Adaptivity to aerodynamic errors