FLIGHT CONTROL DESIGN USING BACKSTEPPING

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Abstract: Today's prevailing nonlinear design method for aircraft flight control is feedback linearization. In this paper, backstepping is proposed as a new method to deal with the nonlinear aerodynamic forces and moments acting on the aircraft. Specifically, backstepping is used to derive state feedback control laws for angle of attack and sideslip control that require less knowledge of the lift and side forces compared to feedback linearization designs. The control laws are shown to be inverse optimal with respect to meaningful cost functionals, which guarantees that stability is preserved for a certain amount of actuator saturation.

Keywords: Aircraft control, backstepping, Lyapunov functions, inverse optimality

1. INTRODUCTION

The development of high performance aircraft operating at high angles of attack and at high angular rates has stimulated the interest in applying nonlinear control techniques to aircraft flight control. Currently, feedback linearization (Isidori, 1995), or nonlinear dynamic inversion (NDI), as it is often referred to in the field, is the prevailing nonlinear design method with numerous applications reported, see, e.g., (Lane and Stengel, 1988), (Enns et al., 1994), (Reiner et al., 1996).

Feedback linearization aims at cancelling the nonlinear system behavior. By using nonlinear feedback, the closed loop system is rendered linear. A drawback with this approach is that for the cancellation to be possible, all the nonlinearities involved must be known exactly. In aircraft flight control, the aerodynamic forces and moments acting on the aircraft are important sources of nonlinearity to be dealt with. In practice these can not be modeled exactly and hence, perfect cancellation is not possible.

In this paper we propose a new approach to robust aircraft flight control. Our main mathematical tool is backstepping (Krstić et al., 1995). Backstepping offers a more flexible way of dealing with nonlinearities compared to feedback linearization. Nonlinearities that act stabilizing may be kept in the closed loop system while destabilizing nonlinearities may be cancelled or dominated.

Using this freedom, we design controllers for angle of attack and sideslip control that do not require complete descriptions of the lift force and the side force respectively. The key is to rely on the generic characteristics of these forces.

To realize the control laws in terms of control surface deflections, the mapping from these deflections to the resulting aerodynamic moment needs to be inverted. Assuming a general mapping between the two, this control allocation problem is solved using nonlinear optimization. Robustness against uncertainties and model errors in the mapping is achieved by recursively estimating the bias from the nominal model and using the estimate for feedback. This can be seen as an alternative to traditional integral feedback.
In this contribution, the controlled variables are the angle of attack, \( \alpha \), the sideslip angle, \( \beta \), and the roll angle, \( \phi \). These depend on the orientation of the aircraft, \( \theta \), and the roll rate about the stability x-axis, \( p \), see Fig. 1. Fig. 1. Illustration of the controlled variables.

2. AIRCRAFT MODEL

In this section, the equations of motion describing the aircraft dynamics in terms of these variables are given by (Boiffier, 1998), (Stevens and Lewis, 1992):

\[
\begin{align*}
\dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta + \frac{1}{mV_T \cos \beta}(-L - F_T \sin \alpha + mg_1) \\
\dot{\beta} &= p \sin \alpha - r \cos \alpha + \frac{1}{mV_T}(Y - F_T \cos \alpha \sin \beta + mg_2) \\
M &= I\dot{\omega} + \omega \times I\omega
\end{align*}
\]

These equations describe the uncontrolled aircraft dynamics when \( \delta \) is set to zero. The body axes angular velocity \( \omega \), is related to the body axes angular velocity \( \dot{\omega} \) through the transformation

\[
\omega_s = S_\alpha \omega, \quad S_\alpha = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}
\]

The control input, \( \delta \), consists of the elevator (\( \delta_e \)), aileron (\( \delta_a \)), and rudder (\( \delta_r \)) deflections.

For backstepping to be applicable, we will assume these control surface deflections only to produce aerodynamic moments, and not forces. We will also neglect the derivatives of the aerodynamic forces with respect to the angular velocity \(^1\).

Essentially, this yields

\[
\begin{align*}
L(\alpha) &= qSC_L(\alpha) \\
Y(\beta) &= qSC_Y(\beta)
\end{align*}
\]

where \( q = \rho V_T^2/2 \) is the aerodynamic pressure, \( \rho \) is the air density, and \( S \) is the wing planform area.

In the control design to come, \( \dot{\omega}_s \), the stability axes angular acceleration, will at first be considered the control input. Introducing

\[
u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \dot{\omega}_s
\]

we can rewrite the aircraft dynamics (1) as

\[
\begin{align*}
\dot{p}_s &= u_1 \\
\dot{q}_s &= \frac{1}{mV_T}(\dot{L}(\alpha) - F_T \sin \alpha + mg_1) \\
\dot{\alpha} &= q_s - p_s \tan \beta + \frac{1}{mV_T \cos \beta}(-L - F_T \sin \alpha + mg_1) \\
\dot{\beta} &= p_s \sin \alpha - r_s \cos \alpha + \frac{1}{mV_T}(Y - F_T \cos \alpha \sin \beta + mg_2) \\
\dot{\gamma}_s &= -r_s \\
\dot{r}_s &= u_3 \\
M &= I\dot{\omega} + \omega \times I\omega
\end{align*}
\]

The relationship between \( u \) and the true control input, \( \delta \), can be found by combining equations (1c), (2), and (3). Regarding \( \alpha \) as a constant while realizing the lateral control demands \( u_1 \) and \( u_3 \) yields \( u = S_\alpha \dot{\omega} \). Inserting this into Eq. (1c) gives us

\[
\begin{align*}
u = S_\alpha F^{-1}(M(\delta) - \omega \times I\omega)
\end{align*}
\]

where we assume \( M \) to be a static function of the demanded control surface deflections, \( \delta \), ignoring the fast actuator dynamics.

Introducing the state vector \( x = (\alpha \beta p_s q_s r_s)^T \) we can use the compact form

\[
\begin{align*}
\dot{x} &= f(x) + Bu \\
u &= g(\delta, x)
\end{align*}
\]

to describe the uncontrolled aircraft dynamics (4)–(5).

\(^1\) These assumptions are the same as in feedback linearization applications.
In (Härkegård and Glad, 2000), it is shown that a stability axis roll, also known as a velocity vector roll, is a roll performed at constant angle of attack and zero sideslip. The sideslip is to be kept zero at all times. Speed control is assumed to be handled separately.

3. CONTROL PRELIMINARIES

3.1 Control objectives

The angle of attack and the stability axis roll rate should follow the pilot commanded values \( \alpha^{ref} \) and \( p_s^{ref} \) respectively. A stability axis roll, also known as a velocity vector roll, is a roll performed at constant angle of attack and zero sideslip. The sideslip is to be kept zero at all times. Speed control is assumed to be handled separately.

3.2 Controller architecture

The block diagram in Fig. 2 gives an overview of the controller architecture that will be used.

The first block, on which this paper focuses, outputs the desired angular acceleration, \( u = k(x) \). If this could be produced exactly by deflecting the control surfaces properly, the closed loop dynamics would be

\[
\dot{x} = f(x) + Bk(x)
\]

In Sections 4.1 and 4.2 we derive state feedback control laws \( k(x) \) such that \( p_s = p_s^{ref} \), \( \alpha = \alpha^{ref} \), and \( \beta = 0 \) becomes a globally asymptotically stable (GAS) equilibrium of the closed loop system (7).

Realizing \( u = k(x) \) requires precise knowledge of which angular acceleration, and in particular which aerodynamic moment, \( M \), is produced for a certain set of control deflections, \( \delta \), see (5). To allow a model error to be present in this usually quite complex mapping, we remodel the system dynamics (6) as

\[
\dot{x} = f(x) + B(u + e) \\
u = \hat{g}(\delta, x)
\]

where \( \hat{g}(\delta, x) \) represents our model of the mapping (5) and \( e \) is the model error, which we will pragmatically model as an unknown but constant bias. Using nonlinear observer techniques (Krener and Isidori, 1983), an exponentially converging estimate, \( \hat{e} \), can be produced. This estimate can be used for feedback in a straightforward way:

\[
u = k(x) - \hat{e}
\]

In (Härkegård and Glad, 2000), it is shown that closed loop stability is preserved using this adaptive control law.

4. CONTROL DESIGN

4.1 Stability axis roll control

Controlling the stability axis roll rate, \( p_s \), is straightforward. Considering its dynamics in Eq. (4a), simply assign

\[
u_1 = k_{p_s}(p_s^{ref} - p_s)
\]

where \( 1/k_{p_s} \) is the desired roll time constant. This corresponds to ordinary proportional control.

4.2 Angle of attack and sideslip control

To begin with, we note the structural similarities between the angle of attack dynamics (4b)–(4c)
Table 1. The relationships between the general nonlinear system (10) and the angle of attack and sideslip dynamics in (4b)–(4e).

<table>
<thead>
<tr>
<th>General system (10)</th>
<th>α dynamics (4b)–(4e)</th>
<th>β dynamics (4d)–(4e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( \alpha^{ref} )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( u )</td>
<td>( q_s )</td>
<td>( -r_s )</td>
</tr>
<tr>
<td>( y )</td>
<td>( p_s, \beta, V_T, h, \theta, \phi )</td>
<td>( \alpha, V_T, h, \theta, \phi )</td>
</tr>
<tr>
<td>( f(x_1, y) )</td>
<td>( f_\alpha(\alpha, y_\alpha) )</td>
<td>( f_\beta(\beta, y_\beta) )</td>
</tr>
</tbody>
</table>

We will first derive a backstepping control law for the generic system (10) and then apply it to \( \alpha \) and \( \beta \) control.

4.2.1. A generic backstepping design Let us consider the system (10), assuming a general nonlinearity \( f \), and determine a control law that makes \( x_1 = x_1^{ref} \) GAS. For convenience, we make the origin the goal state by introducing the new coordinates

\[
\begin{align*}
\xi_1 &= x_1 - x_1^{ref} \\
\xi_2 &= x_2 + f(x_1^{ref}, y) \\
\varphi(\xi_1) &= f(x_1, y) - f(x_1^{ref}, y)
\end{align*}
\]

This yields

\[
\begin{align*}
\dot{\xi}_1 &= \varphi(\xi_1) + \xi_2 \\
\dot{\xi}_2 &= u
\end{align*}
\]

Now assume that there exists a maximum slope

\[
\kappa = \max_{\xi^1_1 \in \mathbb{R}} \frac{\varphi(\xi_1)}{\xi_1} \leq \max_{x_1^{ref} \in \Omega_{ref}} \frac{\partial f(x_1, y)}{\partial x_1}
\]

Equality holds if \( x_1^{ref} \) is not restricted, i.e., when \( \Omega_{ref} = \mathbb{R} \). To use this property in the Lyapunov framework of backstepping, we can rewrite it as

\[
\xi_1 \varphi(\xi_1) \leq \kappa \xi_1^2
\]

We now turn to the actual control design.

Step 1: In the spirit of backstepping, we start by regarding \( \xi_2 \) as the control input of Eq. (13a) and find a desired globally stabilizing “virtual” control law \( \xi_2^{des} \), using the control Lyapunov function (clf)

\[
V_1 = \frac{1}{2} \xi_2^2
\]

Differentiating with respect to time, we get

\[
\dot{V}_1 |_{\xi_2=\xi_2^{des}} = \xi_1(\varphi(\xi_1) + \xi_2^{des}) \leq \xi_1(\kappa \xi_1 + \xi_2^{des})
\]

using (15). \( \dot{V}_1 \) is made negative definite by selecting

\[
\xi_2^{des} = -k_1 \xi_1, \quad k_1 > \kappa
\]

The resulting \( \xi_1 \) dynamics, \( \varphi(\xi_1) - k_1 \xi_1 \), lie in the second and fourth quadrants only and thus, \( \xi_1 \) is stabilized.

Step 2: Continue by introducing the residual

\[
\xi_2 = \xi_2 - \xi_2^{des} = \xi_2 + k_1 \xi_1
\]

and rewrite the system dynamics in terms of \( \xi_1 \) and \( \xi_2 \).

\[
\begin{align*}
\dot{\xi}_1 &= \varphi(\xi_1) - k_1 \xi_1 + \xi_2 \\
\dot{\xi}_2 &= u + k_1(\varphi(\xi_1) - k_1 \xi_1 + \xi_2)
\end{align*}
\]

In Eq. (16b) it is not clear whether the \( \xi_1 \) components are beneficial or not. Proceeding in the usual backstepping manner, by adding a \( \xi_2^{es} \) term to the clf, would lead to a control law that cancels these components. The control law would then contain \( \varphi(\xi_1) \) and consequently require the knowledge of \( f(x_1, y) \) for all \( x_1 \), not only at the equilibrium. As we will see, this can be avoided by also adding
To make the right hand side negative definite, and we can further simplify this expression using our knowledge. In terms of the original state variables in (13), the linear case of the nonlinear nature of the system (13), the backstepping control law (18) minimizes the cost functional

\[ \int_{0}^{\infty} \left( k_1 (\varphi(\xi_1) - k_1 \xi_2) + (\frac{k_2}{2} - k_1)(\xi_2 + k_1 \xi_1) \right)^2 + \frac{1}{2k_2} u^2 ) dt \]

For \( k_2 > 2k_1 \), which is a stricter condition than (20), the penalty on the state variables \( \xi_1 \) and \( \xi_2 \) becomes positive definite. Then the cost functional becomes “meaningful” and the robustness properties of nonlinear control as exploited in (Glad, 1987) hold. This includes a gain margin of \( (k_1/k_2, \infty) \) which guarantees that stability is preserved even when the prescribed input, \( u \), cannot be produced exactly, e.g., due to actuator saturation.

4.2.2. Application to \( \alpha \) and \( \beta \) control Evaluating the control law (19) in the angle of attack control case using Table 1 yields

\[ u_2 = -k_{\alpha,2}(q_\alpha + k_{\alpha,1}(\alpha - \alpha^{\text{ref}}) + f_\alpha(\alpha^{\text{ref}}, y_\alpha)) \]

For inverse optimality to hold, the parameters should be chosen according to

\[ k_{\alpha,2} > 2k_{\alpha,1}, \quad k_{\alpha,1} > \max\{k_{\alpha}, 0\} \]

Since we have put no restrictions on \( \alpha^{\text{ref}} \), evaluating (14) yields

\[ \kappa_\alpha = \max_{\alpha, y_\alpha} \frac{\partial f_\alpha(\alpha, y_\alpha)}{\partial \alpha} \]

Similarly, in the sideslip regulation case we get

\[ u_3 = k_{\beta,2}(-r_\beta + k_{\beta,1}\beta + \frac{q}{V_T} \cos \theta \sin \phi) \]

\[ k_{\beta,2} > 2k_{\beta,1}, \quad k_{\beta,1} > \max\{k_{\beta}, 0\} \]

assuming that \( Y(0) = 0 \). Evaluating (14), using the fact that the sideslip reference is always zero, yields

\[ \kappa_\beta = \max_{\beta, y_\beta} \frac{f_\beta(\beta)}{\beta} \]

\( f_\beta \), defined in (12), is dominated by the side force, \( Y(\beta) \), which resides in the second and fourth quadrants, see Fig. 3. Thus \( \kappa_\beta < 0 \) and the parameter restriction \( k_{\beta,1} > 0 \) becomes active. Note that the \( u_3 \) dependence on the side force is hereby completely removed.

5. SIMULATIONS

We evaluate the control laws using Admire, a MATLAB/SIMULINK environment for the Generic
Aerodata Model (GAM) (Backström, 1997), a small generic fighter aircraft, developed by Saab AB, Sweden. The simulations are performed at an initial speed of 0.5 Mach at an altitude of 1000 m. The control law parameters were set according to $k_p = k_{\alpha,1} = k_{\beta,1} = 2$, $k_{\alpha,2} = k_{\beta,2} = 5$. The poles of the bias observers were placed in $-8 \pm i$. Fig. 4 shows the responses to a sole angle of attack command, a sole stability axis roll command, and the combination of the two. The two lower graphs show the control surface deflections for the combined maneuver along with the bias estimates used in Eq. (8).

6. CONCLUSIONS

In this paper we have demonstrated the potential of using backstepping techniques to design flight control laws. In comparison to feedback linearization, the control laws can be made computationally simpler by not cancelling the beneficial nonlinear parts of the lift and side forces. We have also shown the control laws to solve meaningful optimal control problems which ensures a certain amount of robustness.

7. REFERENCES


