AIAA Guidance, Navigation and Control Conference and Exhibit
20 - 23 August 2007, Hilton Head, South Carolina

AIAA 2007-6421

Vector backstepping design for flight control

Ola Härkegård*
Saab AB, SE-581 88 Linköping, Sweden

Torkel Glad†
Linköping University, SE-581 83 Linköping, Sweden

A backstepping design for an aircraft described by rigid body dynamics is proposed. The system states are the unit velocity vector and the angular velocity in a body-fixed coordinate system. The result can be viewed as a multi-variable controller for angle of attack, sideslip angle, and velocity vector roll rate. The backstepping design is performed on the rigid body dynamics described on vector form. The resulting controller achieves convergence to a desired state from almost all starting points. Guidelines are given for tuning the controller to affect the characteristics of the short period mode, the roll mode and the dutch roll mode. A coupled high angle of attack, high roll rate maneuver is simulated using a simplified aircraft model to illustrate the design.

Nomenclature

V	Velocity vector (body-fixed)
$\hat{V}$	Velocity direction
$\lvert V\rvert$	Total velocity
ω	Angular velocity vector (body-fixed)
α	Angle of attack
β	Angle of sideslip
F	External force
M	External torque
m	Mass, kg
J	Inertia matrix, kg·m^2
$\hat{V}_o$	Reference velocity direction
λ	Reference velocity vector roll rate
W	Control Lyapunov function
K_1, K_2	Control design matrices

I. Introduction

An important tool for nonlinear control synthesis is backstepping.[1–3] The idea is to extend a Lyapunov function from a simple system to systems involving additional state variables and at the same time design the feedback control to guarantee stability.

The purpose of the present paper is to design control laws for a rigid aircraft using backstepping techniques. Backstepping has previously been used to control different types of rigid bodies such as spacecraft,[4] helicopters,[5] marine vessels,[6] and mobile robots.[7] In these papers the controlled variables are the orientation and in some cases also the position of the rigid body. In this paper we consider control of the translational and angular velocities expressed in a body-fixed coordinate system. We will consider motions in three dimensions whereas the designs in ref. 6, 7 consider motions in two dimensions. Further, our model includes external forces, such as aerodynamic or hydrodynamic forces, which is not done in ref. 4, 5, 7.

Current nonlinear designs for flight control often rely on feedback linearization,[8–10] see, e.g., ref. 11–15. In these papers the aircraft model used for control design is described on component form, i.e., in terms of

*Control system engineer, Flight Control System, Saab Aerosystems, ola.harkegard@saab.se
†Professor, Division of Automatic Control, Dept. of Electrical Engineering, torkel@isy.liu.se

individual state variables such as angle of attack, sideslip angle, pitch rate, roll rate, and yaw rate. A key feature of this paper is that backstepping is applied directly the rigid body dynamics expressed on vector form.

The remainder of this paper is organized as follows. In Section II the rigid body aircraft model used for control design is described. Backstepping control design is performed in Section III. This section also contains stability analysis of the resulting closed loop dynamics and some tuning guidelines. The proposed control law is applied to a simple nonlinear aircraft model in Section IV and conclusions are made in Section V.

II. Aircraft dynamics

Let the aircraft be a rigid body with mass m and moment of inertia J. To describe its motion we use a body fixed coordinate system with the origin at the center of mass. The aircraft dynamics is then given by

$$\begin{aligned} m\dot{V} &= -\omega \times mV + F \\ J\dot{\omega} &= -\omega \times J\omega + M \end{aligned} \tag{1}$$

where V is the velocity and ω is the angular velocity. F is the external force resulting from gravity, aerodynamics and engine thrust. M is the external torque due to aerodynamics and possibly also engine thrust.

Assume now that $V \neq 0$. Then the velocity vector may be uniquely written as

$$V = |V|\hat{V} \tag{2}$$

where $|V|$ is the total velocity and $\hat{V}$ has unit length and represents the velocity direction relative to the body. The unit velocity vector can be parameterized as

$$\hat{V} = \begin{pmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \end{pmatrix}^T \tag{3}$$

where α is the angle of attack of the aircraft and β is the sideslip angle.[16] In many applications $|V|$ and $\hat{V}$ are controlled independently of each other. In modern fighter aircraft, such as JAS 39 Gripen, the pilot controls the total velocity $|V|$ by varying the engine thrust, while control of the angle of attack and the sideslip angle (and hence $\hat{V}$) is performed by a flight control system with pilot stick and pedal positions as inputs. It is therefore of interest to model the dynamics of these entities separately.

The dynamics of the total velocity $|V|$ can be derived from the relationship $|V|^2 = V^T V$. Differentiating with respect to time and dividing by $2|V|$ yields

$$\frac{\mathrm{d}}{\mathrm{d}\,t}|V| = \frac{V^T\dot{V}}{|V|} = \frac{1}{m}\hat{V}^T F \tag{4}$$

The dynamics of the velocity direction $\hat{V}$ can be derived by differentiating the relationship $\hat{V} = V/|V|$. After some manipulations, including the use of the relationship

$$F = (\hat{V}^T F)\hat{V} + (\hat{V} \times F) \times \hat{V} \tag{5}$$

this gives

$$\dot{\hat{V}} = \frac{\dot{V}}{|V|} - \frac{V}{|V|^2}\frac{\mathrm{d}}{\mathrm{d}\,t}|V| = \left[-\omega + \frac{1}{m|V|}\hat{V} \times F\right] \times \hat{V} \tag{6}$$

This equation can be viewed as a description of the angle of attack and sideslip dynamics on vector form.

Now assume that the force has the form

$$F = F(V, t)$$

The dependence on V captures the influence from total velocity, angle of attack and sideslip angle on the aerodynamic forces. The dependence on time captures the force contributions from gravity (attitude dependent) and engine thrust if these are viewed as time dependent but known entities. To facilitate the upcoming backstepping design we assume that the force is independent of the angular velocity ω and the available control effectors.

The torque M may depend on, e.g., V, ω, and available control effectors. In this paper we will consider the total torque as a control variable. To emphasize this we introduce $u_M = M$. Transforming a torque command into individual control effector commands is the control allocation problem[17–19] which will not be dealt with here.

In this paper we assume that control of the total velocity $|V|$ is performed by the pilot or some other device and instead focus on the remaining aircraft dynamics. Hence the relevant dynamics are described by

$$\dot{\hat{V}} = \left[-\omega + \frac{1}{m|V|} \hat{V} \times F(V,t) \right] \times \hat{V} \tag{7}$$

$$J\dot{\omega} = -\omega \times J\omega + u_M \tag{8}$$

Let us now investigate what stationary motions are possible for these dynamics. Consider a motion with $\hat{V} = \hat{V}_o$ where $\hat{V}_o$ is a constant unit vector. This corresponds to a motion with $\alpha = \alpha_o$, $\beta = \beta_o$ if α_o, β_o and $\hat{V}_o$ are related as in (3). Inserting this into the velocity equation (7) gives the angular velocity solution

$$\omega = \frac{1}{m|V|} \hat{V}_o \times F(|V|\hat{V}_o, t) + \lambda \hat{V}_o \tag{9}$$

where λ is an arbitrary scalar.

We see that the angular velocity is uniquely determined by $\hat{V}_o$ up to the term $\lambda \hat{V}_o$ which represents a rotation around the velocity vector with angular rate $\omega^T \hat{V}_o = \gamma$, which is the velocity vector roll rate. This angular velocity can be achieved by applying the torque

$$u_M = J\dot{\omega} + \omega \times J\omega$$

according to equation (8).

III. Backstepping control design

In this section we develop a backstepping control design for the model (7)–(8) to steer the velocity direction $\hat{V}$ towards a reference direction $\hat{V}_o$ and the velocity vector roll rate $\omega^T \hat{V}$ towards a reference roll rate λ. The result is a controller which makes

$$\begin{aligned} \hat{V} &= \hat{V}_o \\ \omega &= \frac{1}{m|V|} \hat{V}_o \times F(|V|\hat{V}_o, t) + \lambda \hat{V}_o \end{aligned} \tag{10}$$

an asymptotically stable equilibrium for all starting points outside an exceptional set of lower dimension.

A. Backstepping

STEP 1: We start by considering only the velocity dynamics (7). In the standard backstepping manner we regard ω as a virtual control variable and construct a desired angular velocity ω_d that steers $\hat{V}$ towards $\hat{V}_o$. Consider the control Lyapunov function

$$W_1 = \frac{1}{2} (\hat{V} - \hat{V}_o)^T (\hat{V} - \hat{V}_o)$$

which satisfies

$$\dot{W}_1|_{\omega=\omega_d} = \left[\omega_d - \frac{1}{m|V|} \hat{V} \times F(V,t) \right]^T (\hat{V} \times \hat{V}_o)$$

We now design a virtual control law $\omega = \omega_d$ to make W_1 a decreasing function of time. A possible choice is

$$\omega_d = \frac{1}{m|V|} \hat{V} \times F(V,t) + \lambda \hat{V} - K_1 (\hat{V} \times \hat{V}_o)$$

where $K_1 = K_1^T > 0$. The first terms acts to cancel the effects of the time varying force nonlinearity $F(V,t)$. The second term corresponds to our desire to achieve a velocity vector roll rate of λ. Note that this term could be replaced by a term $\lambda \hat{V}_o$ without affecting $\dot{W}_1|_{\omega=\omega_d}$. It seems more natural however to desire a rotation around the current velocity vector $\hat{V}$ rather than around the steady state velocity vector $\hat{V}_o$. The third term of ω_d adds stability to the $\hat{V}$ dynamics at the desired equilibrium.

This choice of virtual control law leads to

$$\dot{W}_1|_{\omega=\omega_d} = -(\hat{V} \times \hat{V}_o)^T K_1 (\hat{V} \times \hat{V}_o) < 0, \quad \hat{V} \neq \pm \hat{V}_o$$

Note that $\dot{W}_1|_{\omega=\omega_d} = 0$ holds not only at the desired equilibrium $\hat{V} = \hat{V}_o$ but also at $\hat{V} = -\hat{V}_o$.

Step 2: Backstepping through the model (7)–(8) our next task is to design a torque control u_M that steers ω towards ω_d in such a way that the entire system is stabilized around the desired equilibrium (10). Introduce the residual

$$\tilde{\omega} = \omega - \omega_d$$

and the new control variable

$$\bar{u}_M = -\omega \times J\omega + u_M$$

We now have the dynamics

$$\begin{aligned}\dot{\hat{V}} &= (-\tilde{\omega} + K_1(\hat{V} \times \hat{V}_o)) \times \hat{V} \\ J\dot{\tilde{\omega}} &= -J\dot{\omega}_d + \bar{u}_M\end{aligned}$$

To penalize the angular velocity residual $\tilde{\omega}$ we use the extended control Lyapunov function

$$W = \frac{c}{2}(\hat{V} - \hat{V}_o)^T(\hat{V} - \hat{V}_o) + \tilde{\omega}^T J \tilde{\omega}$$

where $c > 0$, which satisfies

$$\begin{aligned}\dot{W} &= c(\hat{V} - \hat{V}_o)^T \left[(-\tilde{\omega} + K_1(\hat{V} \times \hat{V}_o)) \times \hat{V}\right] + \tilde{\omega}^T(-J\dot{\omega}_d + \bar{u}_M) \\ &= -c(\hat{V} \times \hat{V}_o)^T K_1 (\hat{V} \times \hat{V}_o) + \tilde{\omega}^T(c\hat{V} \times \hat{V}_o - J\dot{\omega}_d + \bar{u}_M)\end{aligned}$$

Here we have used the cross product relationship $a^T(b \times c) = b^T(c \times a)$. To make $\dot{W}$ negative we can choose the control

$$\bar{u}_M = -K_2\tilde{\omega} + J\dot{\omega}_d \tag{11}$$

where $K_2 = K_2^T > 0$ gives

$$\begin{aligned}\dot{W} &= -c(\hat{V} \times \hat{V}_o)^T K_1 (\hat{V} \times \hat{V}_o) + c\tilde{\omega}^T(\hat{V} \times \hat{V}_o) - \tilde{\omega}^T K_2 \tilde{\omega} \\ &= -c(K_1(\hat{V} \times \hat{V}_o) - \tfrac{1}{2}\tilde{\omega})^T K_1^{-1} (K_1(\hat{V} \times \hat{V}_o) - \tfrac{1}{2}\tilde{\omega}) \\ &\quad - \tilde{\omega}(K_2 - \tfrac{c}{4}K_1^{-1})\tilde{\omega} \leq 0\end{aligned}$$

if $K_2 - \frac{c}{4}K_1^{-1} > 0$. This condition can be met for all $K_1 > 0$, $K_2 > 0$ by selecting c small enough. Since

$$K_2 - \tfrac{c}{4}K_1^{-1} \geq [\underline{\sigma}(K_2) - \frac{c}{4}\underline{\sigma}(K_1)^{-1}]I_{3\times 3}$$

a sufficient condition on c is

$$0 < c < 4\underline{\sigma}(K_1)\underline{\sigma}(K_2)$$

Further, $\dot{W} = 0$ requires $\tilde{\omega} = 0$ and $\hat{V} = \pm\hat{V}_o$. Let us summarize out backstepping design.

Proposition 1 *Consider the aircraft dynamics* (7)–(8) *and the torque control law*

$$u_M = -K_2(\omega - \omega_d) + J\dot{\omega}_d + \omega \times J\omega \tag{12}$$

where

$$\omega_d = -K_1(\hat{V} \times \hat{V}_o) + \gamma\hat{V} + \frac{1}{m|V|}\hat{V} \times F(V, t) \tag{13}$$

and where $K_1 = K_1^T > 0$, $K_2 = K_2^T > 0$. *Assume that some mechanism keeps the velocity magnitude* $|V|$ *bounded from below and from above. Then the above control law is well defined. A Lyapunov function for the closed loop system is given by*

$$W = \frac{c}{2}(\hat{V} - \hat{V}_o)^T(\hat{V} - \hat{V}_o) + \frac{1}{2}(\omega - \omega_d)^T J(\omega - \omega_d) \tag{14}$$

where $c > 0$ *can be selected so that* $\dot{W} < 0$ *except at the two points* $\hat{V} = \pm\hat{V}_o$, $\omega = \omega_d$.

B. Stability analysis

Applying the torque control (12) gives the closed loop dynamics

$$\dot{\hat{V}} = (-\tilde{\omega} + K_1(\hat{V} \times \hat{V}_o)) \times \hat{V} \tag{15a}$$

$$J\dot{\tilde{\omega}} = -K_2\tilde{\omega} \tag{15b}$$

in terms of the velocity direction $\hat{V}$ and the angular velocity residual $\tilde{\omega}$. We note that the force term $F(V,t)$ is no longer present but has been cancelled using feedback. This term is thus treated the same way it would be treated using feedback linearization. However, unlike feedback linearization the closed loop system above is still nonlinear. This is due to the treatment of the Coriolis force term, which appears also in the closed loop dynamics. We can further note that the dynamics does not depend on the total velocity $|V|$.

The two points that were shown to achieve $\dot{W} = 0$ in the previous section ($\hat{V} = \pm\hat{V}_o$ and $\tilde{\omega} = 0$) both correspond to equilibria of this system. From the invariance principle[10] we know that the system will approach either of these two equilibria as $t \to \infty$. The following stability result holds.

Proposition 2 *Let the assumptions of Proposition 1 be satisfied. Trajectories starting at any initial value will then converge to one of the equilibria $\hat{V} = \hat{V}_o$, $\omega = \omega_d$ or $\hat{V} = -\hat{V}_o$, $\omega = \omega_d$. If the initial value satisfies*

$$W(\hat{V}(0), \omega(0)) < 2c \tag{16}$$

the convergence is always to $\hat{V} = \hat{V}_o$, $\omega = \omega_d$.

Proof 1 *The level set*

$$W(\hat{V}, \omega) \le W(\hat{V}(0), \omega(0)) \tag{17}$$

is bounded for any initial condition $\hat{V}(0), \omega(0)$. It then follows from standard Lyapunov theory,[10] that the system state will converge to a set in (17) where $\dot{W} = 0$, i.e. one of the equilibria $\hat{V} = \hat{V}_o$, $\omega = \omega_d$ or $\hat{V} = -\hat{V}_o$, $\omega = \omega_d$ (Proposition 1). Since $W(-\hat{V}_o, \omega_d) = 2c$ the level set (17) contains only the equilibrium $\hat{V} = \hat{V}_o$, $\omega = \omega_d$ if (16) is satisfied.

The proposition shows that the backstepping control law achieves local asymptotic stability of the desired equilibrium $\hat{V} = \hat{V}_o$, $\omega = \omega_d$. Global asymptotic stability cannot be achieved due to the additional equilibrium point $\hat{V} = -\hat{V}_o$, $\omega = \omega_d$.

C. Linearized dynamics

In this section we further investigate the local stability characteristics of the two equilibria. We do this by linearizing the closed loop dynamics (15) at the two equilibria $V = \pm V_o$, $\tilde{\omega} = 0$. To handle differentiation of cross products we introduce the notation

$$a \times b = S(a)b, \quad S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{18}$$

where S is skew-symmetric, i.e., $S^T = -S$. This leads to the differentiation rules

$$\begin{gathered} \frac{\partial}{\partial b} a \times b = S(a), \quad \frac{\partial}{\partial a} a \times b = S^T(b) \\ \frac{\partial}{\partial a} f(a) \times g(a) = S(f)\frac{\partial g}{\partial a} + S^T(g)\frac{\partial f}{\partial a} \end{gathered} \tag{19}$$

Applying these rules gives the following linearized dynamics around $\hat{V} = -\hat{V}_o$, $\tilde{\omega} = 0$:

$$\frac{\mathrm{d}}{\mathrm{d}\,t} \begin{bmatrix} \hat{V} + \hat{V}_o \\ \tilde{\omega} \end{bmatrix} = \begin{bmatrix} S^T(\hat{V}_o) K_1 S(\hat{V}_o) & -S(\hat{V}_o) \\ 0 & -J^{-1}K_2 \end{bmatrix} \begin{bmatrix} \hat{V} + \hat{V}_o \\ \tilde{\omega} \end{bmatrix} \tag{20}$$

The poles of this system are given by the eigenvalues of the submatrices $A = S^T(\hat{V}_o)K_1S(\hat{V}_o)$ and $-J^{-1}K_2$. The maximum eigenvalue of A satisfies

$$\lambda_{\max}(A) \geq \hat{x}^T A\hat{x} = (\hat{V}_o \times \hat{x})^T K_1(\hat{V}_o \times \hat{x}) > 0, \quad \hat{x} \neq \pm\hat{V}_o$$

This shows that the linearization (20) has at least one unstable pole. Using Lyapunov's indirect method,[10] this implies that $\hat{V} = -\hat{V}_o$, $\tilde{\omega} = 0$ is an unstable equilibrium point of the nonlinear system (15). Hence there will only be a small exceptional set (the stable manifold of the equilibrium) from which convergence to that point is possible.

Similarly, the linearized dynamics around $\hat{V} = \hat{V}_o$, $\tilde{\omega} = 0$ becomes

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\begin{bmatrix}\hat{V} - \hat{V}_o \\ \tilde{\omega}\end{bmatrix} = \begin{bmatrix}-S^T(\hat{V}_o)K_1S(\hat{V}_o) & S(\hat{V}_o) \\ 0 & -J^{-1}K_2\end{bmatrix}\begin{bmatrix}\hat{V} + \hat{V}_o \\ \tilde{\omega}\end{bmatrix} \tag{21}$$

This system can be shown to have one pole at the origin, which reflects that $\hat{V}$ only has two degrees of freedom since its magnitude does not change. The remaining poles are real and strictly negative. This is a limitation of the proposed design since it means that the short period mode, for example, cannot be assigned complex valued poles.

Let us now further investigate how the design matrices K_1 and K_2 affect the closed loop behavior. To this end, we parametrize the velocity direction vector as in (3) and introduce the angular velocity components $\tilde{\omega} = (\tilde{p} \;\; \tilde{q} \;\; \tilde{r})^T$. Considering the common case $\beta_o = 0$, i.e., zero desired sideslip, and selecting the controller parameters as

$$\begin{aligned} K_1 &= \mathrm{diag}(k_\beta,\ k_\alpha,\ k_\beta) \\ K_2 &= J \cdot \mathrm{diag}(k_p,\ k_q,\ k_r) \end{aligned} \tag{22}$$

we can rewrite the closed loop velocity direction dynamics (15a) as

$$\begin{aligned} \dot{\alpha} &= -[k_\alpha \cos\beta + k_\beta(\cos^{-1}\beta - \cos\beta)]\sin(\alpha - \alpha_o) \\ &\quad + \tilde{q} - \tan\beta(\tilde{r}\sin\alpha + \tilde{p}\cos\alpha) \\ \dot{\beta} &= -k_\beta\cos(\alpha - \alpha_o)\sin\beta - \tilde{r}\cos\alpha + \tilde{p}\sin\alpha \end{aligned}$$

Linearizing the dynamics around $\alpha = \alpha_o$, $\beta = 0$, $\tilde{\omega} = 0$ gives the following decoupled longitudinal and lateral dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\begin{bmatrix}\alpha - \alpha_o \\ \tilde{q}\end{bmatrix} = \begin{bmatrix}-k_\alpha & 1 \\ 0 & -k_q\end{bmatrix}\begin{bmatrix}\alpha - \alpha_o \\ \tilde{q}\end{bmatrix} \tag{23}$$

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\begin{bmatrix}\beta \\ \tilde{p} \\ \tilde{r}\end{bmatrix} = \begin{bmatrix}-k_\beta & \sin\alpha_o & -\cos\alpha_o \\ 0 & -k_p & 0 \\ 0 & 0 & -k_r\end{bmatrix}\begin{bmatrix}\beta \\ \tilde{p} \\ \tilde{r}\end{bmatrix} \tag{24}$$

The poles of these systems are given by the negative of the controller parameters k_α, k_β, k_p, k_q, and k_r. These parameters can be tuned by the user to affect the dynamics of the short period mode, the roll mode and the dutch roll mode.

As discussed above, the poles become real independently of the design parameters. This is due to that the lower triangular parts of the system matrices above are zero. In the backstepping design, this can be traced back to the design choice made in (11) where the term $J\dot{\omega}_d$ is cancelled by the control. A possible extension of this work is to investigate if other choices could be made that would allow for less restrictive pole placements.

IV. Flight control example

In this section we illustrate our backstepping design using a simple flight control example.

A. Aircraft model

A simplified version of the ADMIRE fighter aircraft model[20,21] is used for simulation. The main reason for not using the full ADMIRE model is to have the total torque as a control input, rather than the individual control effectors, so that the model can be written as in (7)–(8).

Consider a body-fixed coordinate system with axes pointing forward, over the right wing and down. Let the aircraft mass and moment of inertia be given by

$$m = 9100 \text{ kg}, \quad J = \begin{bmatrix} 21000 & 0 & -2500 \\ 0 & 81000 & 0 \\ -2500 & 0 & 101000 \end{bmatrix} \text{kg}\cdot\text{m}^2$$

Let the force F in (7) be given by

$$F(V,t) = \underbrace{\bar{q}SC_F(\alpha,\beta)}_{\text{aerodynamics}} + \underbrace{mg(\theta(t),\phi(t))}_{\text{gravity}} + \underbrace{F_T(t)\hat{n}}_{\text{engine}}$$

The aerodynamic term is defined by the wing planform area $S = 45 \text{ m}^2$, the dynamic pressure $\bar{q}$ and the aerodynamic force coefficients C_F. The gravitational term depends on the pitch angle $\theta(t)$ and the roll angle $\phi(t)$ of the aircraft. The last term models the effect of an engine producing a thrust force F_T in the body-fixed direction $\hat{n} = (1\ 0\ 0)^T$. For the aerodynamic force coefficients we use the simple model

$$C_F(\alpha,\beta) = -\operatorname{diag}(0.012,\ 0.70,\ 3.5)\hat{V}$$

which has been tuned to resemble the ADMIRE aerodata at low angles of attack. The resulting lift force, side force, and drag force coefficients are shown in Fig. 1. The model captures the basic characteristics of these forces but neglects, for example, the contributions from the angular velocity ω and from the control effectors. However, these contributions are typically small and are often disregarded in nonlinear flight control designs.

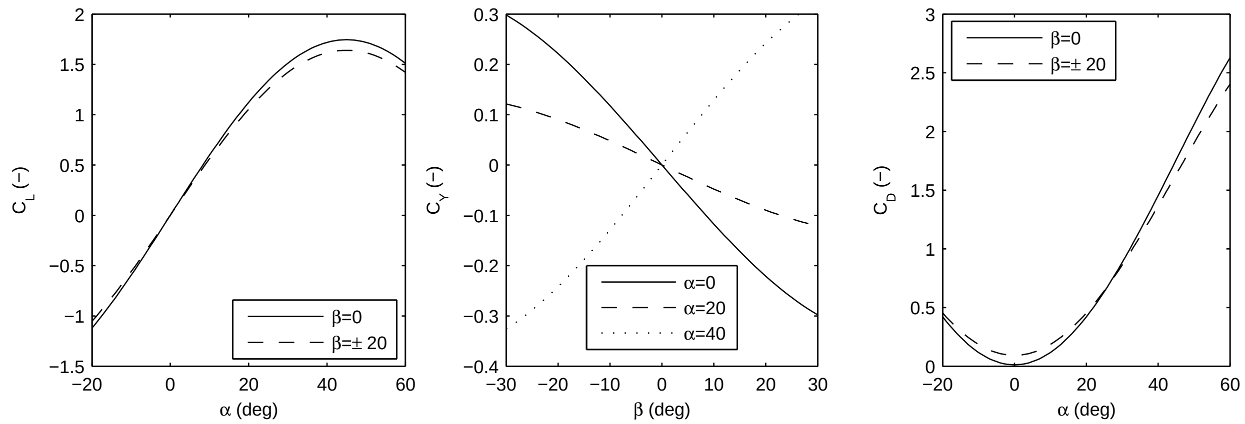

Figure 1. Aerodynamic coefficients for the lift force, side force, and drag force.

B. Controller

To control the aircraft we use the control law in Proposition 1. We select K_1 and K_2 as in (22) with $k_\alpha = k_\beta = 2$, $k_p = k_q = k_r = 2.5$ to obtain reasonable characteristics of the short period mode, the roll mode, and the dutch roll mode.

Implementing the control law (12) is straightforward except for the term $J\dot{\omega}_d$ which requires the expression for ω_d to be differentiated with respect to time. In the current implementation, the first two terms of ω_d are differentiated analytically (considering $\hat{V}_o$ and λ as constants) while the last term is differentiated numerically online.

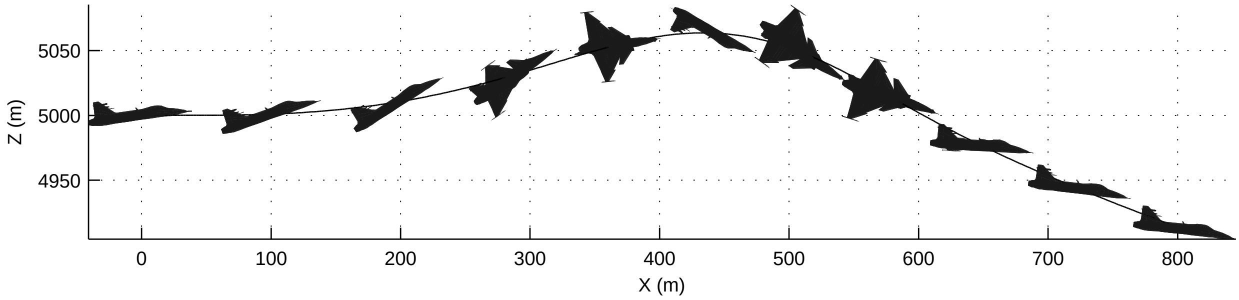

Figure 2. Simulated high angle of attack, high roll rate maneuver.

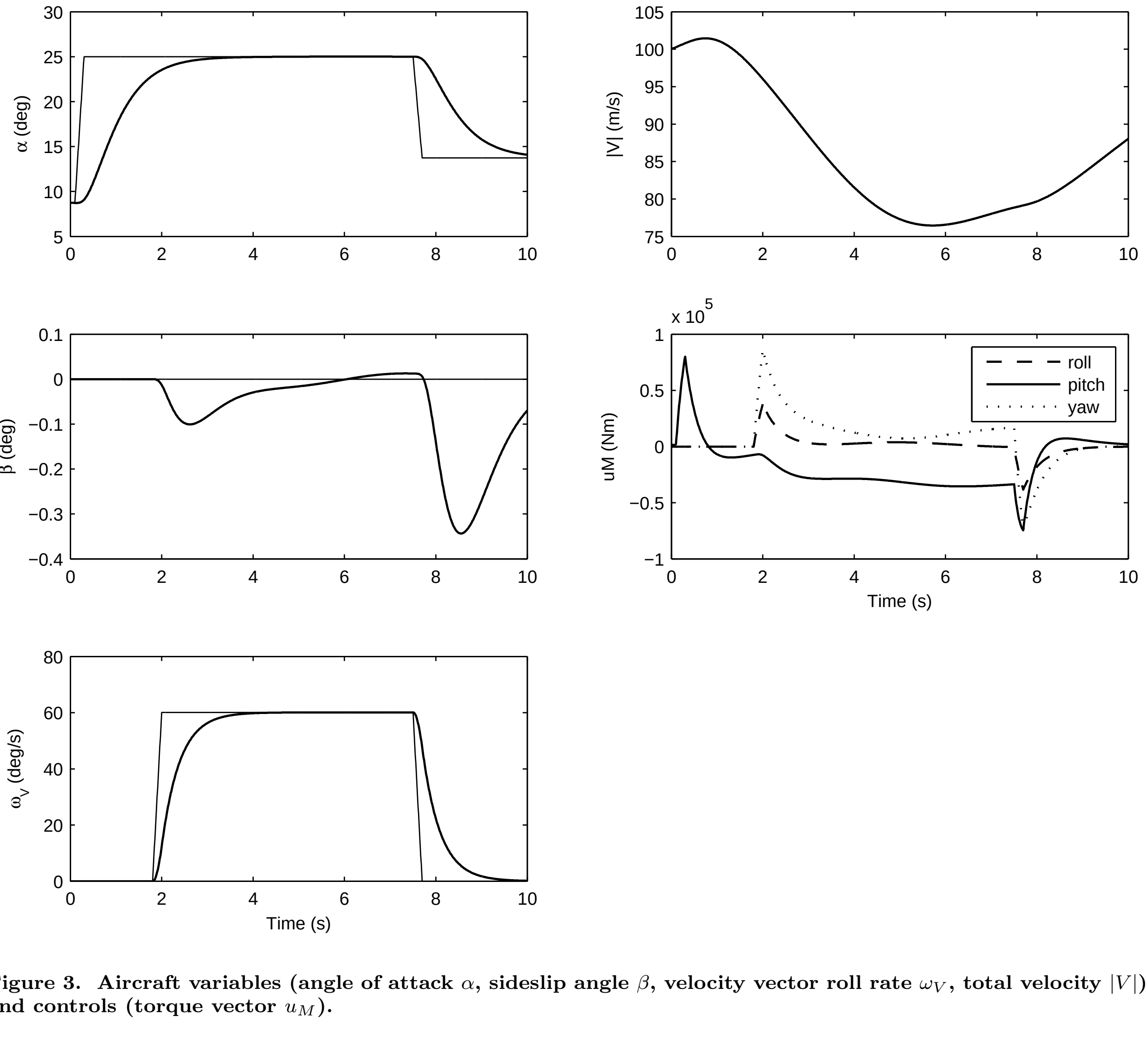

Figure 3. Aircraft variables (angle of attack α, sideslip angle β, velocity vector roll rate ω_V, total velocity $|V|$) and controls (torque vector u_M).

C. Simulations

To illustrate the control laws, a coupled high angle of attack, high roll rate maneuver performed at Mach 0.3, altitude 5000 m is simulated. The maneuver is illustrated in Fig. 2 and plots are shown in Fig. 3. The engine force is held constant at $F_T = 40$ kN.

Initially, a 25 deg angle of attack command is issued, followed by a 60 deg/s velocity vector roll command after 2 seconds. These commands are sustained throughout a 360 deg roll after which both commands are released.

Good response decoupling is achieved throughout the maneuver, the maximum sideslip is less than 0.4 deg. This is of course due to the ideal circumstances for the simulation – perfect measurements, no actuator dynamics, same dynamics used for control design and aircraft simulation, perfect feedforward from roll command to yawing moment etc. Hence, the sideslip that actually does occur is a characteristic of the design. The way the control law steers $\hat{V}$ towards $\hat{V}_o$ does not result in perfect decoupling between α and β even in theory.

V. Conclusions

We have proposed a control law that steers the unit velocity vector and the angular velocity vector (both expressed in a body-fixed coordinate system) of a rigid aircraft to desired values. This can be interpreted as a multi-variable controller for the angle of attack, the sideslip angle, and the velocity vector roll rate, that takes the interaction between these variables automatically into account. The controller achieves convergence to the desired equilibrium from almost all starting points.

A linear analysis showed that the closed loop dynamics, corresponding to the short-period mode, the roll mode, and the dutch roll mode, can be assigned arbitrary but real poles. A possible extension of the present work is to investigate if this restriction can be relaxed by making different choices in the backstepping design process.

References

[1]Krstić, M., Kanellakopoulos, I., and Kokotović, P., *Nonlinear and Adaptive Control Design*, John Wiley & Sons, 1995.

[2]Freeman, R. A. and Kokotović, P. V., *Robust Nonlinear Control Design: State-Space and Lyapunov Techniques*, Birkhäuser, 1996.

[3]Sepulchre, R., Janković, M., and Kokotović, P. V., *Constructive Nonlinear Control*, Springer, Berlin, 1997.

[4]Krstić, M. and Tsiotras, P., "Inverse optimal stabilization of a rigid spacecraft," *IEEE Transactions on Automatic Control*, Vol. 44, No. 5, May 1999, pp. 1042–1049.

[5]Hamel, T. and Mahony, R., "Visual Servoing of an Under-Actuated Dynamic Rigid-Body System: An Image-Based Approach," *IEEE Transactions on Robotics and Automation*, Vol. 18, No. 2, April 2002, pp. 187–198.

[6]Fossen, T. I. and Berge, S. P., "Nonlinear vectorial backstepping design for global exponential tracking of marine vessels in the presence of actuator dynamics," *Proc. of the 36th IEEE Conference on Decision and Control*, Dec. 1997, pp. 4237–4242.

[7]Jiang, Z.-P. and Nijmeijer, H., "Tracking Control of Mobile Robots: A Case Study in Backstepping," *Automatica*, Vol. 33, No. 7, 1997, pp. 1393–1399.

[8]Isidori, A., *Nonlinear Control Systems*, Springer, 3rd ed., 1995.

[9]Slotine, J.-J. E. and Li, W., *Applied Nonlinear Contol*, Prentice Hall, 1991.

[10]Khalil, H. K., *Nonlinear Systems*, Prentice-Hall, 3rd ed., 2002.

[11]Lane, S. H. and Stengel, R. F., "Flight Control Design Using Non-linear Inverse Dynamics," *Automatica*, Vol. 24, No. 4, 1988, pp. 471–483.

[12]Enns, D., Bugajski, D., Hendrick, R., and Stein, G., "Dynamic inversion: an evolving methodology for flight control design," *International Journal of Control*, Vol. 59, No. 1, Jan. 1994, pp. 71–91.

[13]Adams, R. J., Buffington, J. M., and Banda, S. S., "Design of Nonlinear Control laws for High-Angle-of-Attack Flight," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 4, 1994, pp. 737–746.

[14]Reiner, J., Balas, G. J., and Garrard, W. L., "Flight Control Design Using Robust Dynamic Inversion and Time-scale Separation," *Automatica*, Vol. 32, No. 11, 1996, pp. 1493–1504.

[15]Escande, B., "Nonlinear dynamic inversion and LQ techniques," *Robust Flight Control: A Design Challenge*, edited by J.-F. Magni, S. Bennani, and J. Terlouw, chap. 32, Springer, 1997, pp. 523–540.

[16]Stevens, B. L. and Lewis, F. L., *Aircraft Control and Simulation*, John Wiley & Sons, Hoboken, NJ, 2nd ed., 2003.

[17]Durham, W. C., "Constrained Control Allocation," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 4, July–Aug. 1993, pp. 717–725.

[18]Bodson, M., "Evaluation of optimization methods for control allocation," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, July–Aug. 2002, pp. 703–711.

[19]Härkegård, O., *Backstepping and Control Allocation with Applications to Flight Control*, PhD thesis no. 820, Department of Electrical Engineering, Linköping University, May 2003.

[20]Swedish Defence Research Agency (FOI), "Aerodata Model in Research Environment (ADMIRE)," http://www.foi.se/admire/.
[21]Backström, H., "Report on the usage of the Generic Aerodata Model," Tech. rep., Saab Aircraft AB, May 1997.