CONTROL OF SYSTEMS WITH INPUT NONLINEARITIES AND UNCERTAINTIES: AN ADAPTIVE APPROACH

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Abstract

Although many of today's nonlinear control design algorithms assume the system dynamics to be affine in the control input, this often does not hold in practice. A remedy for this is to instead design a control law in terms of some other entity that satisfies the structural assumptions of the design method. In this contribution we discuss how to realize such a virtual control law in terms of the actual control variable. Given a nominal static invertible model of the mapping between the two, the true mapping is assumed to differ from the model by a constant bias. Two ways of how to estimate this bias on-line and use it for feedback are proposed. One of them corresponds to adaptive backstepping, the other one is an observer based approach. In both cases we investigate how to guarantee closed loop stability when the estimate is used for feedback.

1 Introduction

Many of today's constructive nonlinear control design methods assume the system dynamics to be affine in the control input, i.e., for the model to be of the form

$$\dot{x} = f(x) + g(x)u$$

In many practical cases this is not true. A common solution, see, e.g., [1, 2, 6], is to find some other entity, a virtual control input v, in which the dynamics are affine, and that depends on the true control input u through a static mapping. Using, e.g., backstepping or feedback linearization, a globally stabilizing control law $v = k(x)$ can then be derived. These virtual control inputs are often physical entities like forces, torques, or flows, while the true input might be the deflection of a control surface in a flight control case or the throttle setting in an engine control case.

The remaining problem, how to find which actual control input u to apply, is often very briefly discussed, typically assuming that the mapping from u to v is completely known and invertible. In this paper we investigate the case where the mapping is only partially known. It might be that the true mapping is too complex to identify, or that other sources than u contribute to v. Friction might for example reduce the net torque in a robot control case. Here, we will pragmatically model the discrepancy between the model and the true mapping as a constant bias. We propose two different ways of adapting to the bias, and for each case, the issue of closed loop stability is investigated.

2 Problem Formulation

Consider a nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x) + Bv \\ v &= g(x, u) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the measurable state vector, $u \in \mathbb{R}^m$ is the true control input, and $v \in \mathbb{R}^l$, $l \leq n$ is the virtual control input in which the system dynamics are affine. B is an $n \times l$ constant matrix with rank l[1].

From a preceding control design, a control law

$$v = k(x) \tag{2}$$

is assumed to be known such that $x = 0$ is a globally asymptotically stable (GAS) equilibrium [3] of

$$\dot{x} = f(x) + Bk(x)$$

We also assume that a Lyapunov function $V(x)$ for the closed loop system is known, such that

$$\dot{V}(x) = V_x(x)(f(x) + Bk(x)) = -W(x) \tag{3}$$

where $W(x)$ is positive definite.

Now, assume that the mapping $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^l$ from the true control input, u, to the virtual control input, v, is not completely known but only a model, $\hat{g}$, such that

$$g(x, u) = \hat{g}(x, u) + \tilde{g}(x, u)$$

We will assume that the nominal model is invertible in the sense that for each $x \in \mathbb{R}^n$, $w \in \mathbb{R}^l$ there exists a $u \in \mathbb{R}^m$ which solves

$$\hat{g}(x, u) = w$$

[1] This assumption ensures that the number of virtual control inputs is not redundant.

This means that we disregard, e.g., actuator saturation.

If we let u_0 denote the (unknown) control input that solves $f(0) + Bg(0, u_0) = 0$, the model error can be split into

$$\tilde{g}(x, u) = \theta + \delta(x, u)$$

where

$$\theta = \tilde{g}(0, u_0)$$
$$\delta(x, u) = \tilde{g}(x, u) - \tilde{g}(0, u_0)$$

Here, $\theta \in \mathbb{R}^l$ is a constant bias while $\delta(x, u)$ by construction vanishes at the desired equilibrium. Disregarding the model error and implementing

$$\hat{g}(x, u) = k(x)$$

would give us the closed loop system

$$\dot{x} = f(x) + Bk(x) + B(\theta + \delta(x, u))$$

Since θ in general will be non-zero, the desired equilibrium property of the origin would then be lost.

Previous work on how to deal with this type of model error, which enters at the same point as the control input, includes Lyapunov redesign and nonlinear damping, see [3]. Here, the basic idea is to augment the nominal control law with an additional state feedback term to dominate the effects of the model error. A weakness of these methods is that they either lead to chattering in the control input or that only boundedness of the solutions can be guaranteed, and not asymptotic stability.

Here, we will focus on achieving asymptotic stability using a continuous dynamic control law, adapting to and cancelling the effects of the nonvanishing perturbation, θ. Also, due to the difficulties in analyzing the effects of $\delta(x, u)$, we will pragmatically assume the control design to be robust against this perturbation and disregard it in our analysis to come.

With this we can rewrite (1) as

$$\dot{x} = f(x) + B(w + \theta) \tag{4a}$$
$$w = \hat{g}(x, u) \tag{4b}$$

w is the part of the virtual control input, v, that we are truly in control of.

Given θ, (2) could be realized by solving

$$w = v - \theta \iff \hat{g}(x, u) = k(x) - \theta \tag{5}$$

for u. How do we deal with the fact that θ is not available? A straightforward solution is to rely on one of the corner stones of adaptive control and use the certainty equivalence [5] of (5). This means that we replace the unknown parameter vector θ by an estimate $\hat{\theta}$ and form

$$w = \hat{g}(x, u) = k(x) - \hat{\theta} \tag{6}$$

Figure 1 illustrates the approach. The strategy is intuitively appealing but leads to two important questions:

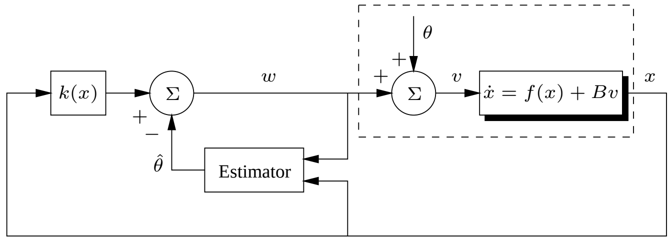

Figure 1: Illustration of the certainty equivalence controller (6). The shadowed box represents the dynamic system to be controlled. The dashed box makes up the system that the control designer actually faces, since θ is not known.

- How do we estimate θ?
- Can we retain global stability using $\hat{\theta}$ for feedback?

Two approaches to the problem will be pursued. In Section 3, we will use standard adaptive backstepping techniques to find an estimator that will guarantee closed loop stability without having to adjust the control law (6). In Section 4, the starting point is that a converging estimator is given. The question then is how to adjust the control law to retain stability.

3 Adaptive Backstepping

Adaptive backstepping [5] deals with the unknown parameter vector θ by extending the Lyapunov function $V(x)$ with a term penalizing the estimation error $\tilde{\theta} = \theta - \hat{\theta}$:

$$V_a(x, \tilde{\theta}) = V(x) + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

where

$$\Gamma = \begin{pmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_l \end{pmatrix} \tag{7}$$

is a matrix containing adaptation gains $\gamma_i > 0$ for $1 \leq i \leq l$.

By cleverly selecting the update rule

$$\dot{\hat{\theta}} = \tau(x, \hat{\theta})$$

closed loop stability can be guaranteed. To see this we investigate $\dot{V}_a$ when (6) is used as feedback. Using (3) and (6) we get

$$\begin{aligned} \dot{V}_a &= V_x(x)(f(x) + B(k(x) - \hat{\theta} + \theta)) - \tau(x, \hat{\theta})^T \Gamma^{-1} \tilde{\theta} \\ &= -W(x) + (V_x(x)B - \tau(x, \hat{\theta})^T \Gamma^{-1})\tilde{\theta} \end{aligned} \tag{8}$$

The first term is negative definite according to the assumptions, while the second, mixed term is indefinite. Since $\tilde{\theta}$ is not available, the best we can do is to cancel the second term by selecting

$$\tau(x, \hat{\theta}) = \tau(x) = \Gamma(V_x(x)B)^T \tag{9}$$

The resulting closed loop system becomes

$$\begin{aligned} \dot{x} &= f(x) + B(k(x) + \tilde{\theta}) \\ \dot{\tilde{\theta}} &= -\Gamma(V_x(x)B)^T \end{aligned} \tag{10}$$

which satisfies

$$\dot{V}_a(x, \tilde{\theta}) = -W(x)$$

Despite $\dot{V}_a$ only being negative semidefinite, the origin, $x = 0,\ \tilde{\theta} = 0$, is GAS. This follows from the fact that only $x = 0$ solves $\dot{V}_a = 0$ and that for x to remain zero, $B\tilde{\theta} = 0$ must also hold. Since B has rank l according to the assumptions, this implies $\tilde{\theta} = 0$. Thus, $x = 0$, $\tilde{\theta} = 0$ is a GAS equilibrium according to LaSalle's invariance principle [3].

The following proposition summarizes the preceding discussion.

Proposition 1 (Adaptation using backstepping)
Consider the system

$$\dot{x} = f(x) + B(w + \theta)$$

where $x \in \mathbb{R}^n$ is the measurable state vector, $w \in \mathbb{R}^l$, $l \leq n$ is the control input, $\theta \in \mathbb{R}^l$ is an unknown constant vector, and B in an $n \times l$ matrix with rank l.

Let $w = k(x) - \theta$ be a control law such that $x = 0$ becomes a GAS equilibrium and let $V(x)$ be a Lyapunov function for the closed loop system such that

$$\dot{V}(x) = V_x(x)(f(x) + Bk(x)) < 0, \quad x \neq 0$$

Then, the control law

$$w = k(x) - \hat{\theta} \tag{11a}$$

$$\dot{\hat{\theta}} = \Gamma(V_x(x)B)^T \tag{11b}$$

with Γ as is (7)*, renders $x = 0$, $\hat{\theta} = \theta$ a GAS equilibrium.* □

4 Observer Based Adaptation

In the previous section, the estimator was a consequence of assigning a negative Lyapunov function time derivative. In this section, we first design an estimator and then investigate how to possibly adjust the certainty equivalence control law (6).

4.1 The general case

The idea is to regard θ as an unknown but constant state vector. Augmenting the original dynamics (4a) with this extra state vector yields

$$\begin{aligned} \dot{x} &= f(x) + B(w + \theta) \\ \dot{\theta} &= 0 \end{aligned} \tag{12}$$

Although this system is nonlinear, we can design an observer for θ with linear error dynamics, since the nonlinearity, $f(x)$, is a function of the measurable states only, see [4]. A nonlinear observer is given by

$$\frac{d}{dt}\begin{pmatrix} \hat{x} \\ \hat{\theta} \end{pmatrix} = \begin{pmatrix} f(x) + B(w + \hat{\theta}) \\ 0 \end{pmatrix} + \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}(x - \hat{x}) \tag{13}$$

The dynamics of the estimation error $\epsilon = \begin{pmatrix} x - \hat{x} & \theta - \hat{\theta} \end{pmatrix}^T$ become linear:

$$\dot{\epsilon} = \begin{pmatrix} -K_1 & B \\ -K_2 & 0 \end{pmatrix}\epsilon = A_\epsilon \epsilon \tag{14}$$

To show that the eigenvalues of A_ϵ can be placed arbitrarily, we examine the linear parts of (12). $\xi = \begin{pmatrix} x & \theta \end{pmatrix}^T$ has the linear internal dynamics

$$\dot{\xi} = \begin{pmatrix} 0 & B \\ 0 & 0 \end{pmatrix}\xi = A_\xi \xi$$

and the output equation is given by

$$y = x = \begin{pmatrix} I_n & 0 \end{pmatrix}\xi = C_\xi \xi$$

This yields the observability matrix

$$\mathcal{O} = \begin{bmatrix} C_\xi \\ C_\xi A_\xi \\ \vdots \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ 0 & B \\ \vdots & \vdots \end{bmatrix}$$

with rank $n + l$ according to the rank assumption on B. Thus the error dynamics of the observer can be chosen arbitrarily.

Let us now determine a Lyapunov function for the observer. For any asymptotically stabilizing observer gains K_1 and K_2, making A_ϵ Hurwitz[2], we can find a positive definite matrix P such that

$$\frac{d}{dt}\epsilon^T P \epsilon = -\epsilon^T \underbrace{\begin{pmatrix} I_n & 0 \\ 0 & \Gamma^{-1} \end{pmatrix}}_{Q} \epsilon \leq -\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{15}$$

with Γ as in (7), by solving the Lyapunov equation

$$A_\epsilon^T P + P A_\epsilon = -Q$$

according to basic linear systems theory [7].

To investigate the closed loop stability, we combine the original Lyapunov function $V(x)$ with $\epsilon^T P \epsilon$ and form

$$V_o(x, \epsilon) = V(x) + \epsilon^T P \epsilon$$

We also augment the control law (6) with an extra term, l, to be decided, to compensate for using $\hat{\theta}$ for feedback. The resulting control law

$$w = k(x) + l(x, \hat{\theta}) - \hat{\theta} \tag{16}$$

[2] A matrix is Hurwitz if all its eigenvalues are in the open left half plane.

yields

$$\dot{V}_o = V_x(x)(f(x) + B(k(x) + l(x,\hat{\theta}) - \hat{\theta} + \theta)) - \epsilon^T Q\epsilon$$
$$\leq -W(x) + V_x(x)B(l(x,\hat{\theta}) + \tilde{\theta}) - \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$$

By choosing

$$l(x,\hat{\theta}) = l(x) = -\Gamma(V_x(x)B)^T \tag{17}$$

we can perform a completion of squares.

$$\begin{aligned}\dot{V}_o &\leq -W(x) - V_xB\Gamma(V_xB)^T + V_xB\tilde{\theta} - \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\\ &= -W(x) - \frac{3}{4}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\\ &\quad - \big[(V_xB)^T - \frac{1}{2}\Gamma^{-1}\tilde{\theta}\big]^T\Gamma\big[(V_xB)^T - \frac{1}{2}\Gamma^{-1}\tilde{\theta}\big]\\ &< 0, \quad x \neq 0,\ \tilde{\theta} \neq 0\end{aligned} \tag{18}$$

Thus, the control law (16) in combination with the observer (13) makes $x = 0, \hat{\theta} = \theta$ a GAS equilibrium.

Before we conclude, let us study a special case where (6) does not need to be augmented with $l(x)$ in order for closed loop stability to be guaranteed.

4.2 The optimal control case

Let us consider the case where the original, unattainable control law (2) solves an optimal control problem of the form

$$\min_v \int_0^\infty (q(x) + v^T R(x)v)dt \tag{19}$$

Here, $q(x)$ is a positive definite function and $R(x)$ is a symmetric positive definite matrix satisfying

$$v^T R(x)v \leq v^T R_0 v \text{ for all } x \in \mathbb{R}^n \tag{20}$$

where R_0 is a constant symmetric positive definite matrix. Then the control law can be expressed as

$$k(x) = -\frac{1}{2}R^{-1}(x)(V_xB)^T$$

for some Lyapunov function $V(x)$ solving the corresponding Hamilton-Jacobi-Bellman equation, see [8]. Furthermore, V satisfies

$$\dot{V} = -W(x) = -q(x) - \frac{1}{4}V_xBR^{-1}(x)(V_xB)^T$$

when the optimal control law is used.

Selecting $\Gamma = R_0^{-1}$ in (15) and using the certainty equivalence control law (6) yields (c.f. Equation (18))

$$\begin{aligned}\dot{V}_o &\leq -q(x) - \frac{1}{4}V_xBR^{-1}(x)(V_xB)^T + V_xB\tilde{\theta} - \tilde{\theta}^T R_0\tilde{\theta}\\ &= -q(x) - \tilde{\theta}^T(R_0 - R(x))\tilde{\theta}\\ &\quad - \big[\frac{1}{2}(V_xB)^T - R(x)\tilde{\theta}\big]^T R^{-1}(x)\big[\frac{1}{2}(V_xB)^T - R(x)\tilde{\theta}\big]\\ &< 0, \quad x \neq 0,\ \tilde{\theta} \neq 0\end{aligned}$$

Hence, in this case we do not need to augment the certainty equivalence control law (6) with an extra state feedback term in order to guarantee global stability. An intuitive interpretation of this result is that some of the optimal control effort can be sacrificed in order to compensate for using the estimate $\hat{\theta}$ for feedback.

4.3 Main result

Let us summarize our discussion.

Proposition 2 (Observer based adaptation)
Consider the system

$$\dot{x} = f(x) + B(w + \theta)$$

where $x \in \mathbb{R}^n$ is the measurable state vector, $w \in \mathbb{R}^l$, $l \leq n$ is the control input, $\theta \in \mathbb{R}^l$ is an unknown constant vector, and B in an $n \times l$ matrix with rank l.

Let $w = k(x) - \theta$ be a control law such that $x = 0$ becomes a GAS equilibrium and let $V(x)$ be a Lyapunov function for the closed loop system such that

$$\dot{V}(x) = V_x(x)(f(x) + Bk(x)) < 0, \quad x \neq 0$$

Then, the controller

$$w = k(x) - \Gamma(V_x(x)B)^T - \hat{\theta} \tag{21a}$$

$$\frac{d}{dt}\begin{pmatrix}\hat{x}\\ \hat{\theta}\end{pmatrix} = \begin{pmatrix}f(x) + B(w + \hat{\theta})\\ 0\end{pmatrix} + \begin{pmatrix}K_1\\ K_2\end{pmatrix}(x - \hat{x}) \tag{21b}$$

with Γ as is (7)*, and where $\begin{pmatrix}-K_1 & B\\ -K_2 & 0\end{pmatrix}$ is Hurwitz, renders $x = 0, \hat{\theta} = \theta$ a GAS equilibrium.*

Furthermore, if $v = w + \theta = k(x)$ solves an optimal control problem of the form

$$\min_v \int_0^\infty (q(x) + v^T R(x)v)dt$$

where $R(x)$ satisfies (20)*, then the above result also holds when* (21a) *is replaced by the certainty equivalence control law*

$$w = k(x) - \hat{\theta} \tag{22}$$

□

5 A Water Tank Example

Let us apply the two strategies to a practical example to investigate their pros and cons. Consider the two tanks in Figure 2. The control objective is to achieve a certain water level r in the lower tank. Using Bernoulli's equation and setting all constants to unity, the system dynamics become

$$\begin{matrix}\dot{x}_1 = -\sqrt{x_1} + v\\ \dot{x}_2 = -\sqrt{x_2} + \sqrt{x_1}\end{matrix} \iff \dot{x} = \underbrace{\begin{pmatrix}-\sqrt{x_1}\\ -\sqrt{x_2} + \sqrt{x_1}\end{pmatrix}}_{f(x)} + \underbrace{\begin{pmatrix}1\\ 0\end{pmatrix}}_{B}v$$

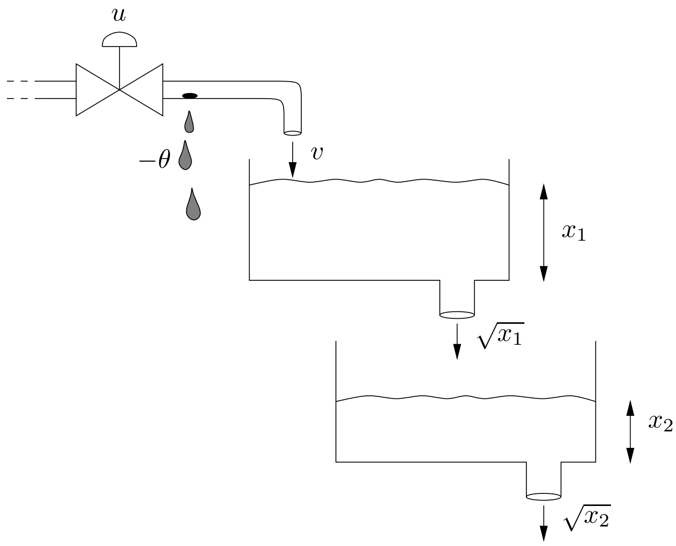

Figure 2: Two tanks connected in series.

where x_1 = water level of the upper tank, x_2 = water level of the lower tank, and v = incoming water flow. v is produced by changing the aperture, u of the valve of the input pipe.

Assuming some external water supply to keep a constant pressure, v will be proportional to the aperture opening area, which in turn depends on u^2. Again setting all constants to unity yields $v = u^2$. In order to be able to account for a possible model error in this static relationship, e.g., due to a leakage $-\theta > 0$, we assign the model

$$v = u^2 + \theta = w + \theta$$

in accordance with (4a).

The first step is to find a globally stabilizing control law $v = k(x)$. We do this using an ad hoc Lyapunov approach. At the desired steady state, $x_1 = x_2 = r$. Therefore consider the control Lyapunov function

$$V(x) = \frac{1}{2}(x_1 - r)^2 + \frac{a}{2}(x_2 - r)^2, \quad a > 0$$

and compute its time derivative:

$$\dot{V}(x) = (x_1 - r)(-\sqrt{x_1} + k(x)) + a(x_2 - r)(-\sqrt{x_2} + \sqrt{x_1})$$

By collecting the beneficial terms and cancelling the indefinite ones, one finds that

$$\begin{aligned} k(x) &= \sqrt{r} + b(r - x_1) + \frac{a}{\sqrt{x_1} + \sqrt{r}}(r - x_2) \\ a &> 0, b \geq 0 \end{aligned} \tag{23}$$

yields $\dot{V}(x) = -W(x)$ where

$$\begin{aligned} W(x) &= (x_1 - r)(\sqrt{x_1} - \sqrt{r}) + b(x_1 - r)^2 \\ &\quad + a(x_2 - r)(\sqrt{x_2} - \sqrt{r}) \end{aligned}$$

is positive definite.

Let us now evaluate the expressions involved with the two approaches for adapting to the leakage. The adaptive backstepping update rule (11b) for estimating θ becomes

$$\dot{\hat{\theta}} = \gamma_a \frac{\partial V(x)}{\partial x_1} = \gamma_a (x_1 - r), \quad \gamma_a > 0 \tag{24}$$

With this, the implicit control law (11a) becomes

$$w = u^2 = k(x) + \gamma_a \int_0^t (r - x_1(s))ds \tag{25}$$

Hence, using adaptive backstepping in this case corresponds to adding integral action from control error in the upper tank.

Using the observer based approach, the estimator can be designed according to (21b), but since only x_1 is directly affected by θ we can use the reduced observer

$$\begin{aligned} \frac{d}{dt}\begin{pmatrix} \hat{x}_1 \\ \hat{\theta} \end{pmatrix} &= \begin{pmatrix} -\sqrt{x_1} + w + \hat{\theta} \\ 0 \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}(x_1 - \hat{x}_1) \\ &= \begin{pmatrix} -k_1 & 1 \\ -k_2 & 0 \end{pmatrix}\begin{pmatrix} \hat{x}_1 \\ \hat{\theta} \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} x_1 + \begin{pmatrix} 1 \\ 0 \end{pmatrix}(-\sqrt{x_1} + w) \end{aligned} \tag{26}$$

where the latter formulation is more suited for implementation. It is also interesting to compute the input-output description of the observer, which is given by

$$\begin{aligned} \hat{\theta} &= \frac{k_2}{s^2 + k_1 s + k_2}(s x_1 - (-\sqrt{x_1} + w)) \\ &\approx \dot{x}_1 - (-\sqrt{x_1} + w) = \theta \end{aligned}$$

for low values of $s = i\omega$. This means that the observer can be interpreted as computing the difference between $\dot{x}_1$ and the expected dynamics, $-\sqrt{x_1} + w$.

The implicit control law (21a) becomes

$$w = u^2 = k(x) + \gamma_o (r - x_1) - \hat{\theta}, \quad \gamma > 0$$

If $b > 0$ is selected in the control law (23), we do not have to add the term $\gamma_o(r - x_1)$ for the sake of stability, since it can be seen as a part of $k(x)$ already. As in the optimal control case treated in Section 4.2, closed loop is then guaranteed using the original certainty equivalence control law (22) without any modification.

In the simulations, the parameter values were selected according to Table 1. The initial water level, which is also fed to the observer, is 1 in both tanks. The control objective is for x_2 to reach the reference level $r = 4$ and maintain this despite the leakage $\theta = -3$ starting at $t = 25$ s. Figure 3 shows the actual control input and the water level of the lower tank when no adaptation is used. Figures 4 and 5 show the results of applying adaptive backstepping and observer based adaptation, respectively. The leakage estimates in the two cases are shown in Figure 6.

The main difference between the two adaptation schemes is that the backstepping update rule (24) depends explicitly on the control error while the observer estimate from (26) evolves independently of the control error. This can be seen clearly in the initial behavior of the two estimates in Figure 6.

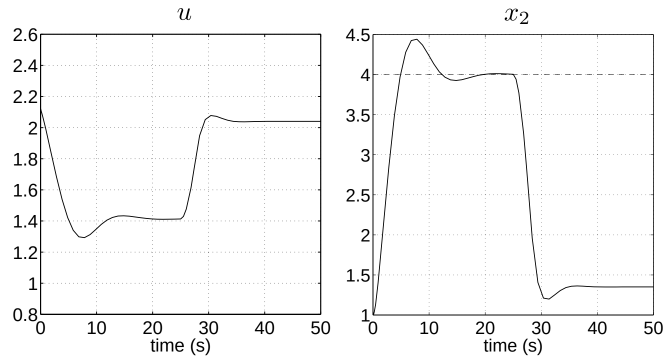

Figure 3: No adaptation, pure state feedback.

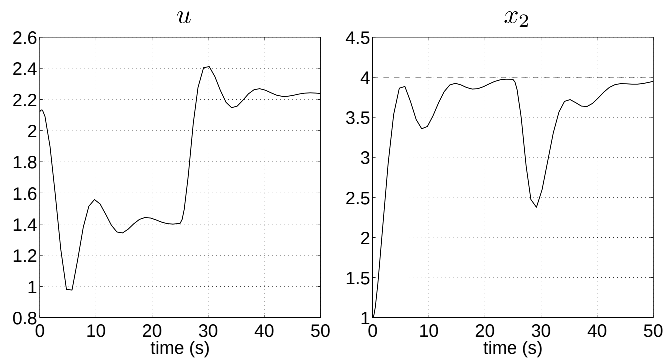

Figure 4: Adaptive backstepping.

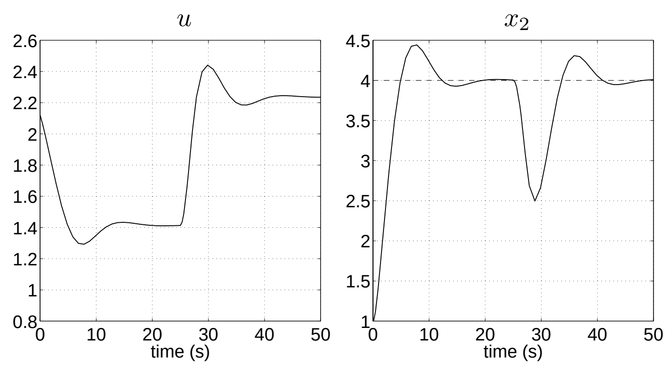

Figure 5: Observer based adaptation.

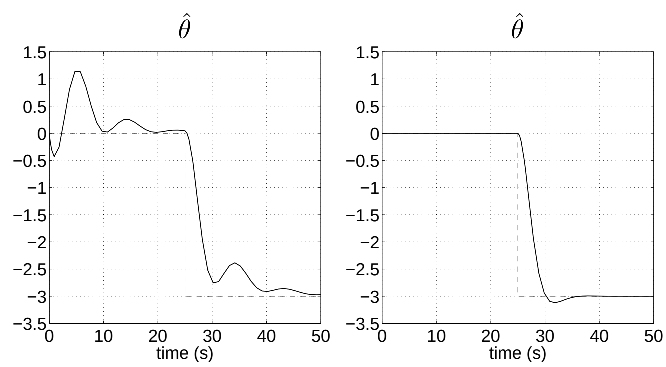

Figure 6: Bias estimates using adaptive backstepping (left) and observer based adaptation (right).

$k(x)$	Adaptive backstepping	Observer based adaptation
$a = 1$ $b = 0.5$	$\gamma_a = 0.3$	$k_1 = 1$ $k_2 = 0.5$ $\gamma_o = 0$

Table 1: Controller parameter values used in the simulations.

6 Conclusions

In this paper we have proposed an adaptive approach to the problem of controlling systems with input nonlinearities and uncertainties. The mapping from the true control input to some virtual control input, in which the system dynamics are affine, was modeled as a known static invertible mapping plus an unknown bias. The intuitively attractive idea of estimating the bias and compensating for it in the feedback law proved to work also in theory. Two paths were investigated. First, adaptive backstepping was applied, and was found to correspond to adding integral action in the considered example. Second, an observer based approach was taken, resulting in a slight change of the original feedback law to account for using the estimated bias for feedback. However, if the original feedback law is optimal, closed loop stability is guaranteed without changing the feedback, due to its inherent gain margin.

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