Backstepping Control of a Rigid Body

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Abstract

A method for backstepping control of rigid body motion is proposed. The control variables are torques and the force along the axis of motion. The proposed control law and lyapunov function guarantee asymptotic stability from all initial values except one singular point.

1 Introduction

An important tool for nonlinear control synthesis is backstepping, see e.g. [4], [8]. The idea is to extend a Lyapunov function from a simple system to systems involving additional state variables and at the same time design the feedback control to guarantee stability. This technique has been successful in several applications, [1, 2, 3, 9]. Recently backstepping design has been successfully applied to the control of aircraft, [5, 6, 7]. The aircraft dynamics is essentially described by rigid body dynamics in combination with equations describing the aerodynamic forces. There are several ways of designing controllers for rigid body equations occuring in various applications, see e. g. [10, 11]. The purpose of the present paper is to formulate a design method for a controlled rigid body using backstepping techniques. The design can then be specialized to aircraft control problems or the control of various types of vehicles.

2 Rigid body dynamics

We assume that the controlled object is a rigid body with mass m. We describe the motion in a body fixed coordinate system with the origin at the centre of mass and obtain the equations:

$$\begin{aligned} \dot{V} &= -\omega \times V + \frac{1}{m}F \\ I\dot{\omega} &= -\omega \times I\omega + M \end{aligned} \tag{1}$$

where V is the velocity, ω is the angular velocity, F is the external force and M is the external torque (all these quantities are vectors with three components). I is the moment of inertia. We will assume that the force has the form

$$F = m(F_a(V) + u_v\hat{V})$$

where $\hat{V} = \frac{1}{|V|}V$ and u_v is a control variable. The first part, F_a, corresponds to aerodynamic or hydrodynamic forces, and the second part models approximately the thrust action of an engine aligned with the velocity vector. The moment M is assumed to depend on V, ω and control variables.

3 Stationary motion

Consider a motion with $V = V_o$, $\omega = \omega_o$ where V_o, ω_o are constants. The velocity equation is then

$$\omega_o \times V_o = F_a(V_o) + u_v\hat{V}_o$$

where $\hat{V}_o = \frac{1}{|\hat{V}_o|}V_o$. Multiplying with $\hat{V}_o$ shows that u_v has to satisfy

$$u_v = -\hat{V}_o^T F_a(V_o)$$

Then ω_o can be calculated from

$$\omega_o = \frac{1}{|V_o|}(\hat{V}_o \times F_a(V_o)) + \gamma\hat{V}_o$$

where γ is an arbitrary constant. If u_v and M can be chosen arbitrarily it is thus possible to achieve a stationary motion for any value of V_o.

4 Backstepping design

In this section we develop a backstepping design to make V_o, ω_o a stable equilibrium. Define

$$u_M = I^{-1}(M - \omega \times I\omega)$$

We will regard u_M as the control signal. Then the dynamics is given by

$$\begin{aligned} \dot{V} &= -\omega \times V + F_a(V) + u_v\hat{V} \\ \dot{\omega} &= u_M \end{aligned} \tag{2}$$

First regard the angular velocity ω (together with u_v) as the control variable. Let V_0 be the desired velocity vector and introduce the Lyapunov candidate

$$W_1 = \frac{1}{2}(V - V_o)^T(V - V_o)$$

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Choose a control of the form

$$\omega = \omega^{\text{des}} = \bar{\omega} + \frac{1}{|V|^2} V \times F_a(V)$$

After some manipulations this gives

$$\dot{W}_1 = -\bar{\omega}^T (V_o \times V) + \bar{u}_v (V - V_o)^T \hat{V}$$

where $u_v = \bar{u}_v - \hat{V}^T F_a$. Choosing

$$\bar{\omega} = k_1 (V_o \times V), \quad \bar{u}_v = -(V - V_o)^T \hat{V}$$

then gives

$$\omega^{\text{des}} = k_1 (V_o \times V) + \frac{1}{|V|^2} V \times F_a(V)$$

$$\dot{W}_1|_{\omega=\omega^{\text{des}}} = -k_1 |V_0 \times V|^2 - ((V - V_o)^T \hat{V})^2 \leq 0$$

In this expression $\dot{W}_1|_{\omega=\omega^{\text{des}}} = 0$ only if $V = V_o$ (provided the singularity $V = 0$ is avoided, which can be done e.g. by starting so that $|V - V_o| < |V_o|$). The lyapunov function thus guarantees convergence to the desired $V = V_o$.

Define

$$\xi = \omega - \omega^{\text{des}}$$

In the new variables the dynamics is

$$\dot{V} = -\xi \times V + k_1 (|V|^2 V_0 - (V^T V_0) V) + \bar{u}_v \hat{V}$$
$$\dot{\xi} = u_M + \phi(V, \xi)$$

where $\phi(V, \xi) = \frac{d}{dt}(\omega^{\text{des}})$. Introducing

$$W_2 = k_2 W_1 + \frac{1}{2} \xi^T \xi$$

gives

$$\dot{W}_2 = -k_1 k_2 |V_0 \times V|^2 - k_2 ((V - V_o)^T \hat{V})^2 - k_3 \xi^T \xi \leq 0$$

if we select the control

$$u = k_2 V_0 \times V - \phi - k_3 \xi$$

Since $\dot{W}_2 = 0$ only occurs for $V = V_o$, $\xi = 0$ (except for the singular case $V = 0$, discussed above) there will be convergence to $V = V_o$, $\xi = 0$.

5 Conclusions

We have proposed a control law that steers the velocity and angular velocity vectors to desired values. The control law uses external torques and a force along the velocity vector. This configuration is similar to, but not precisely equal to the one used in aircraft control, where control surfaces generate torques and the engine gives a longitudinal force. However, our proposed rigid body control could inspire new aircraft control designs. An interesting extension would be to take the orientation into account, which would make it possible to e.g. include the effect of forces like gravity.

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